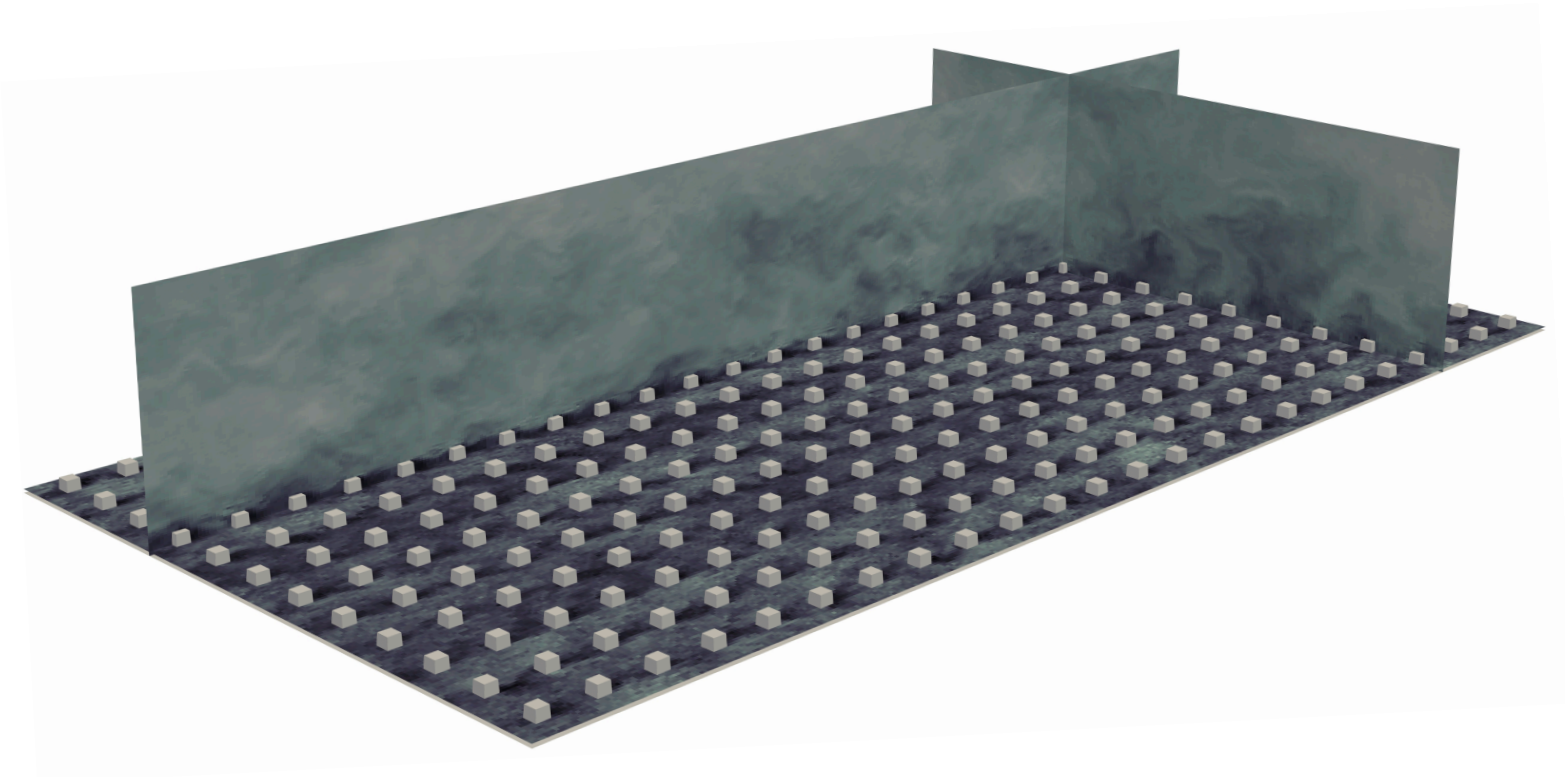


Impact of numerical domain on turbulent flow statistics: scalings and considerations for canopy flows



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Examples of different domains in the literature.

Claus et al. (2012):

$$L_z/h = 4$$

$$L_x : L_y : L_z = 2 : 2 : 1$$

Coceal et al. (2006):

$$L_z/h = 4$$

$$L_x : L_y : L_z = 1 : 1 : 1$$

Schmid et al. (2019):

$$L_z/h = 4$$

$$L_x : L_y : L_z = 1.5 : 1.5 : 1$$

Xie and Castro (2006):

$$L_z/h = 10$$

$$L_x : L_y : L_z = 1.6 : 1.6 : 1$$

Stroh et al. (2020):

$$L_z/h = 23.25$$

$$L_x : L_y : L_z = 8 : 4 : 1$$

Yang and Anderson (2017):

$$L_z/h = 15$$

$$L_x : L_y : L_z = \pi : \pi : 1$$

Cheng and Porte-Agel (2015):

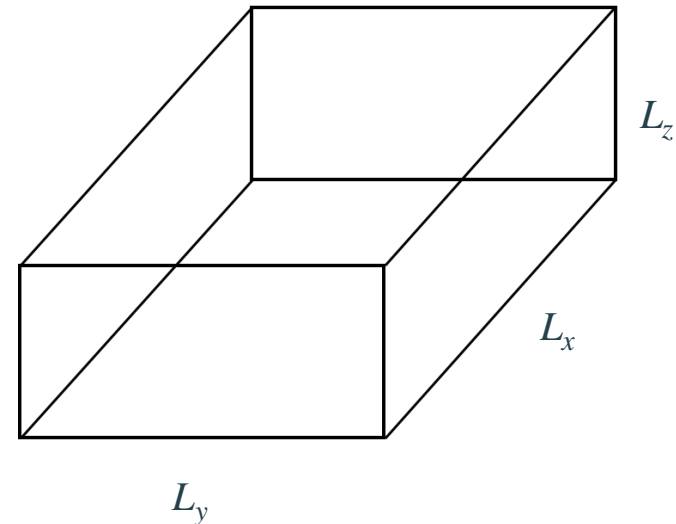
$$L_z/h = 14$$

$$L_x : L_y : L_z = 2.28 : 0.85 : 1$$

Leonardi and Castro (2010):

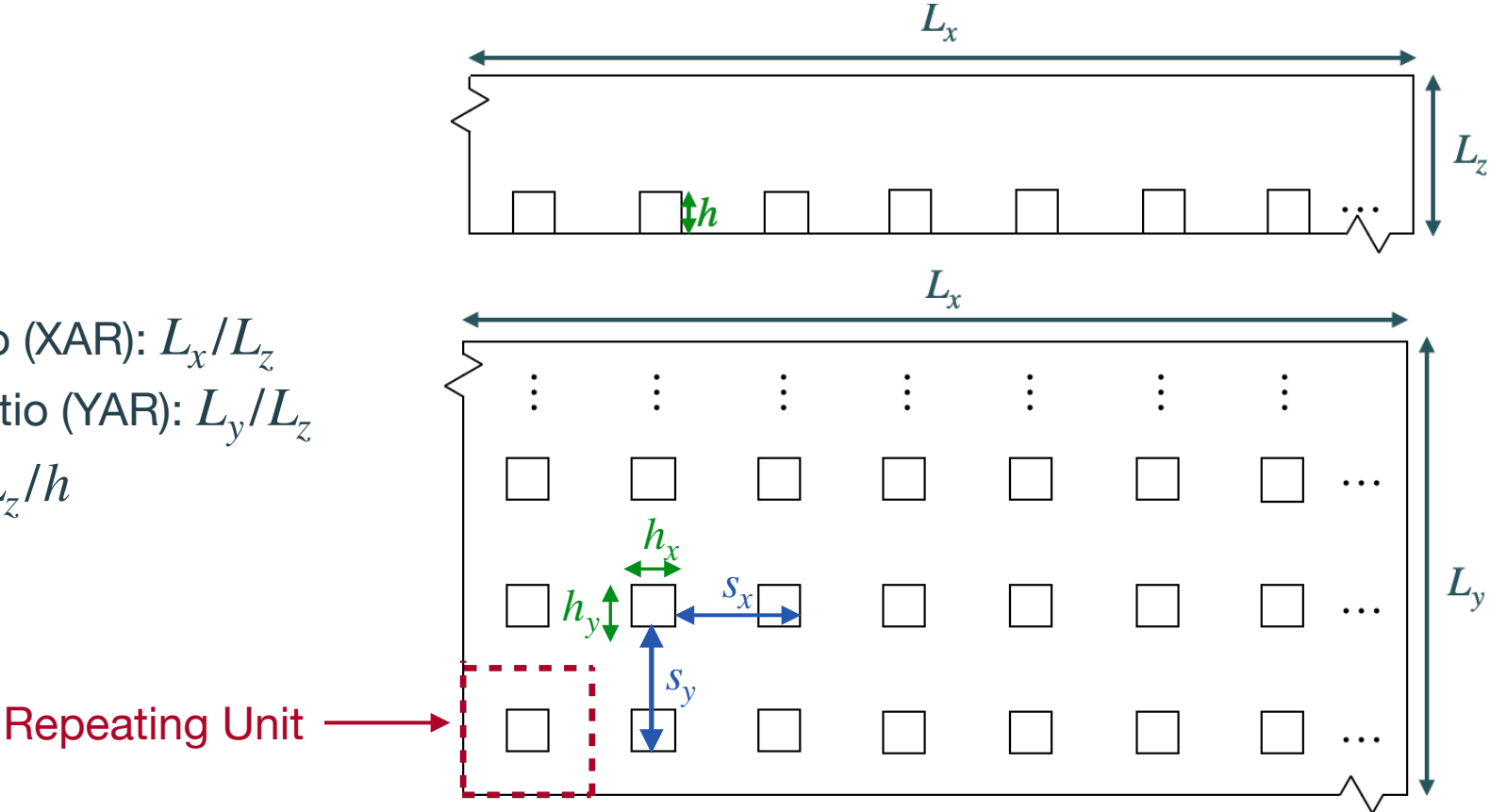
$$L_z/h = 8$$

$$L_x : L_y : L_z = 1 : 0.75 : 1$$



Surface geometry consists of cuboids arranged in square configuration.

Streamwise Aspect ratio (XAR): L_x/L_z
Cross-stream Aspect ratio (YAR): L_y/L_z
Scale separation (SS): L_z/h



Surface geometry consists of cuboids arranged in square configuration.

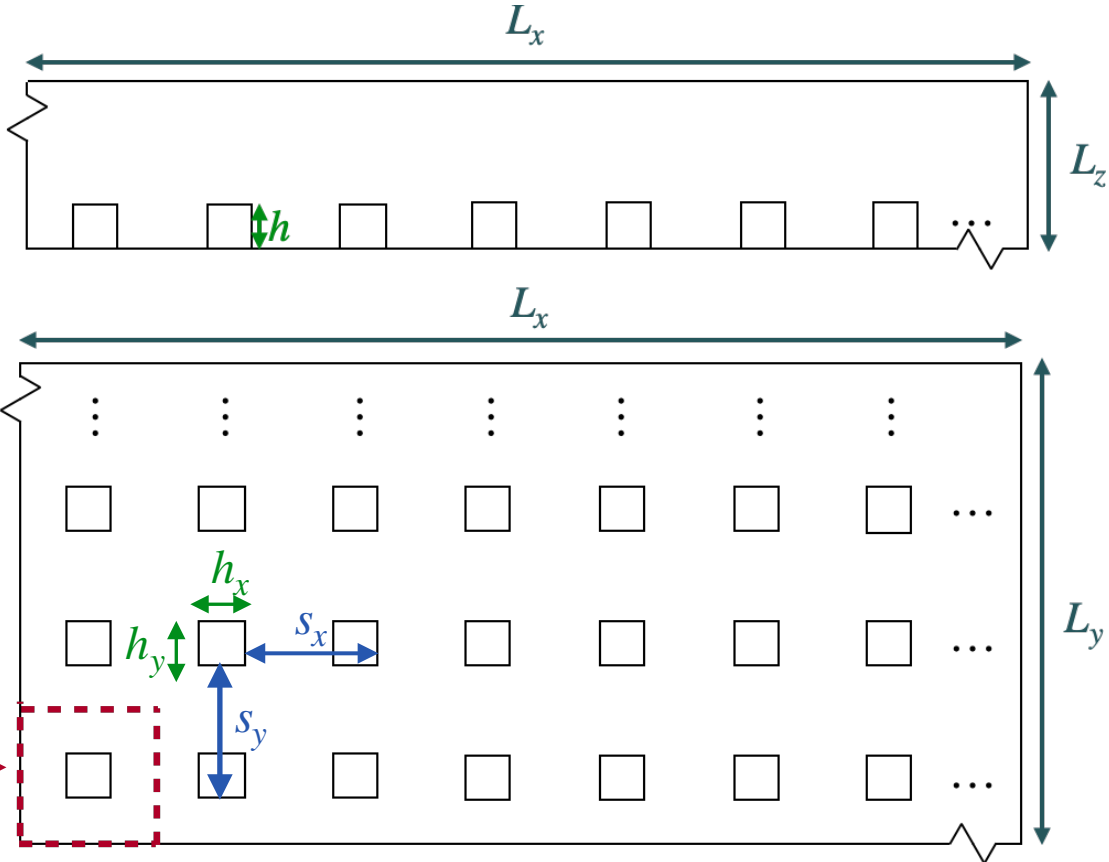
Streamwise Aspect ratio (XAR): L_x/L_z

Cross-stream Aspect ratio (YAR): L_y/L_z

Scale separation (SS): L_z/h

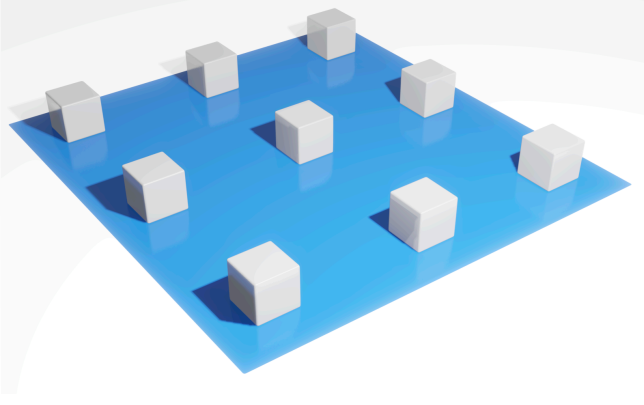
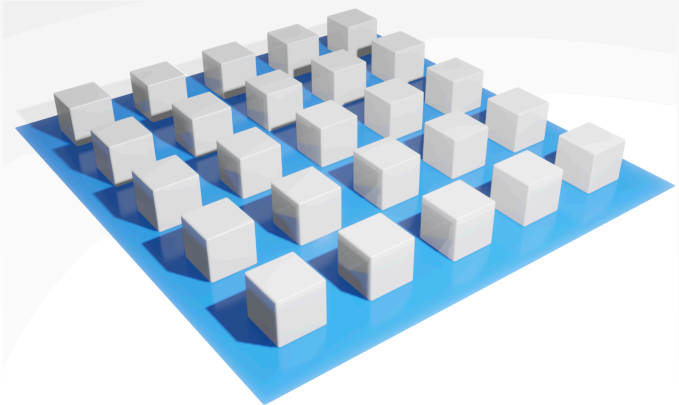
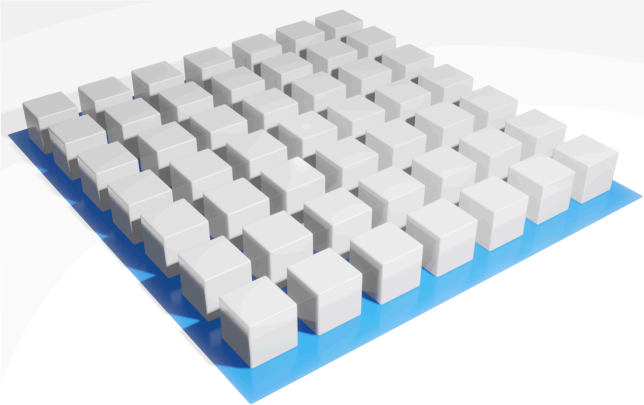
4, 8, 12 and 16

Repeating Unit →

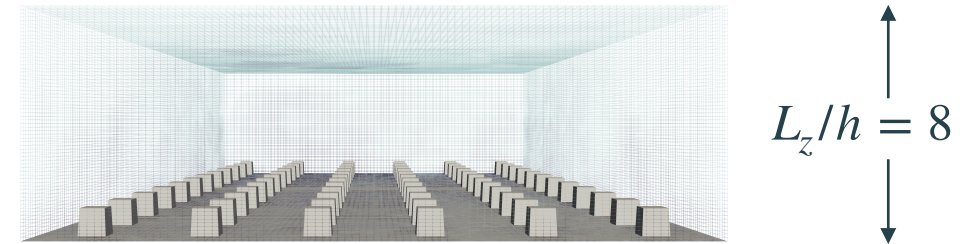
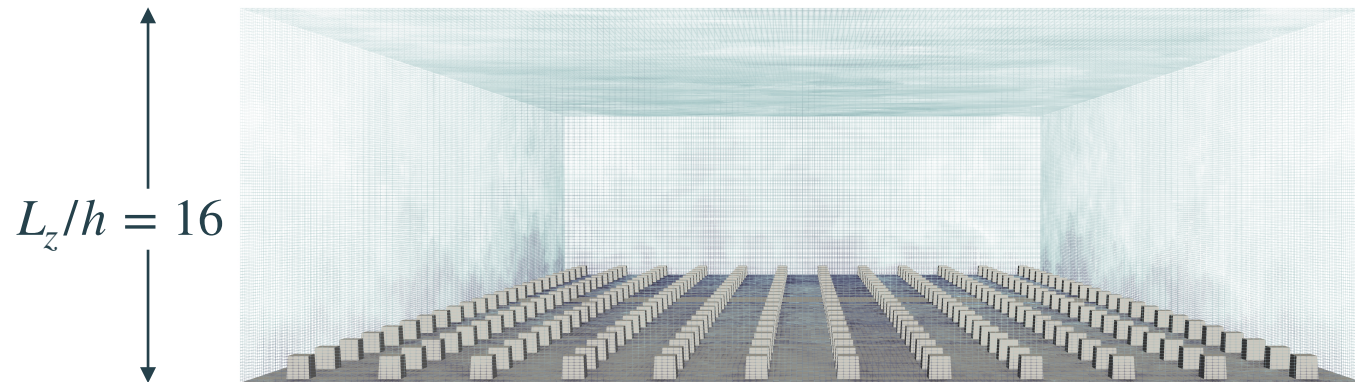


Packing density of the surface is systematically varied.

Decreasing packing density (λ)



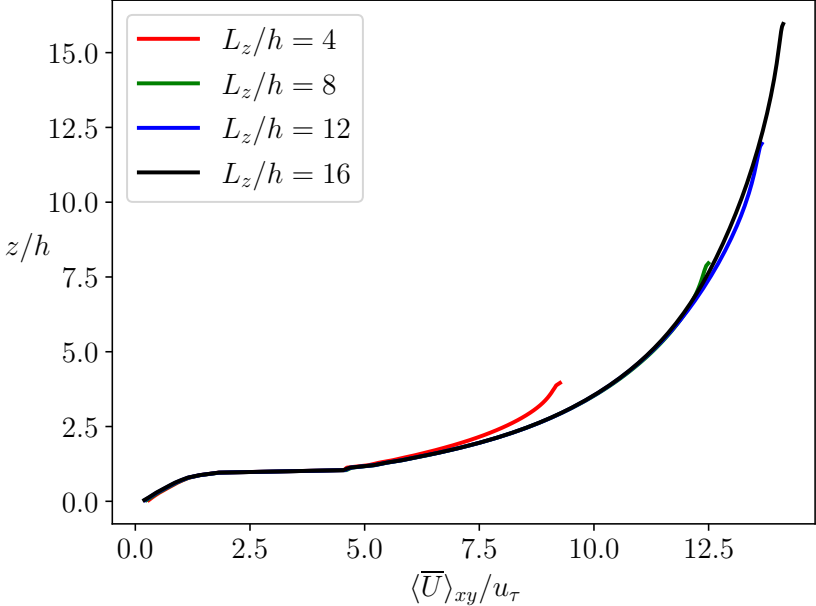
Visualization of cases with different scale separations.



Streamwise and cross-stream aspect ratio are kept constant at 6 and 3 respectively.

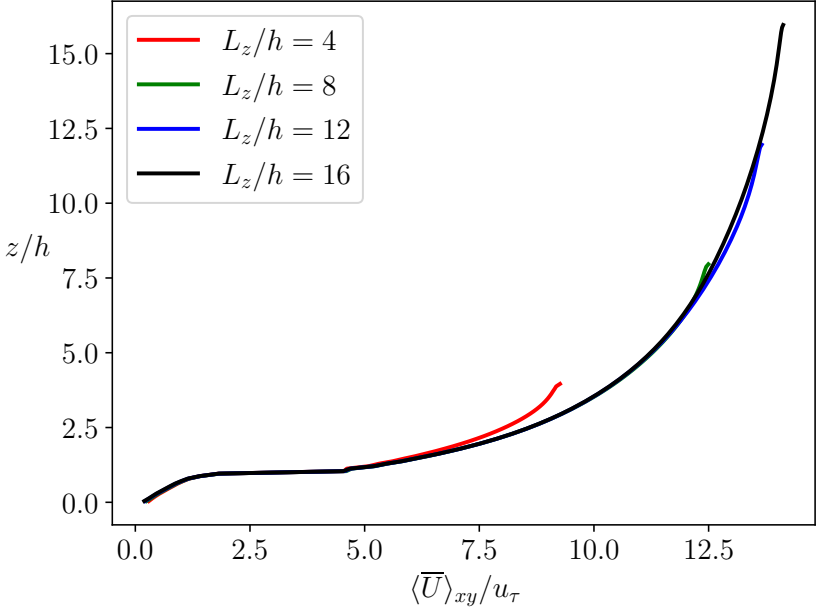
Collapse for mean streamwise velocity is observed for dense configuration;

Dense, $\lambda = 0.25$

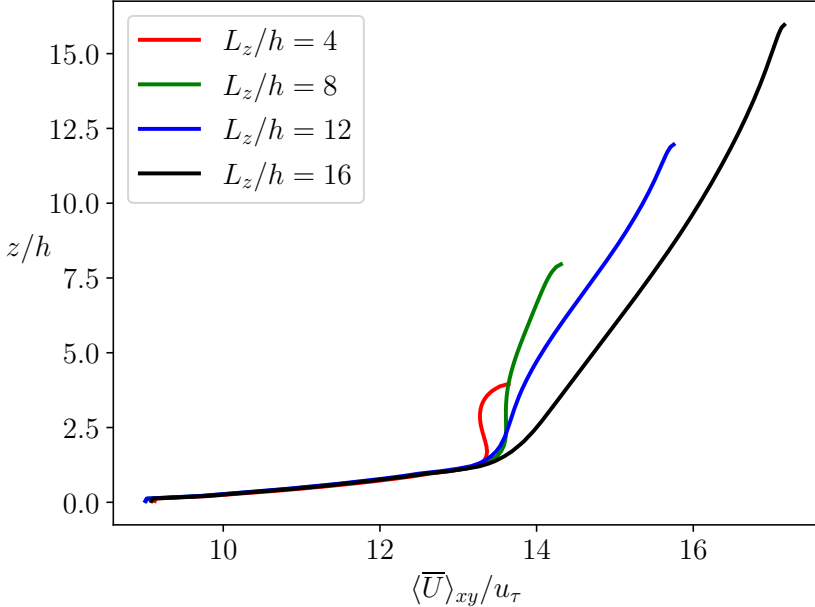


Collapse for mean streamwise velocity is observed for dense configuration; **but not for sparse?**

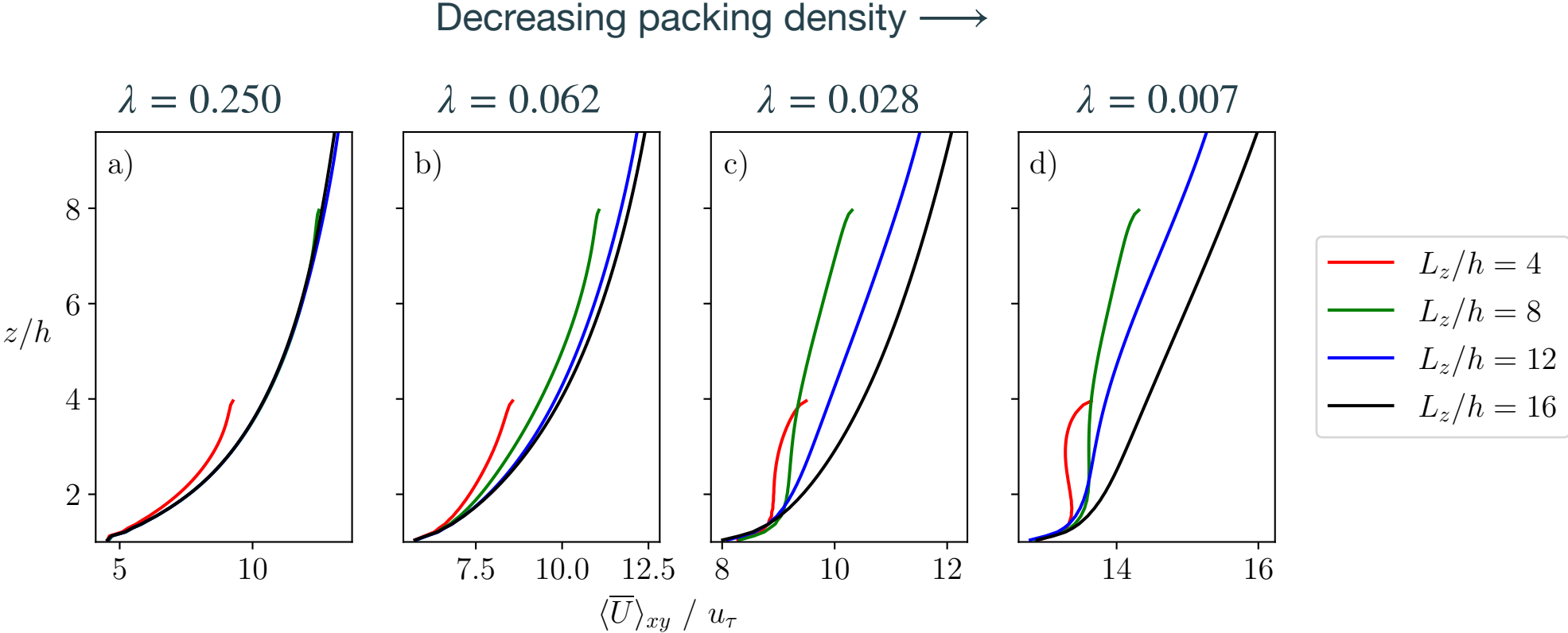
Dense, $\lambda = 0.25$



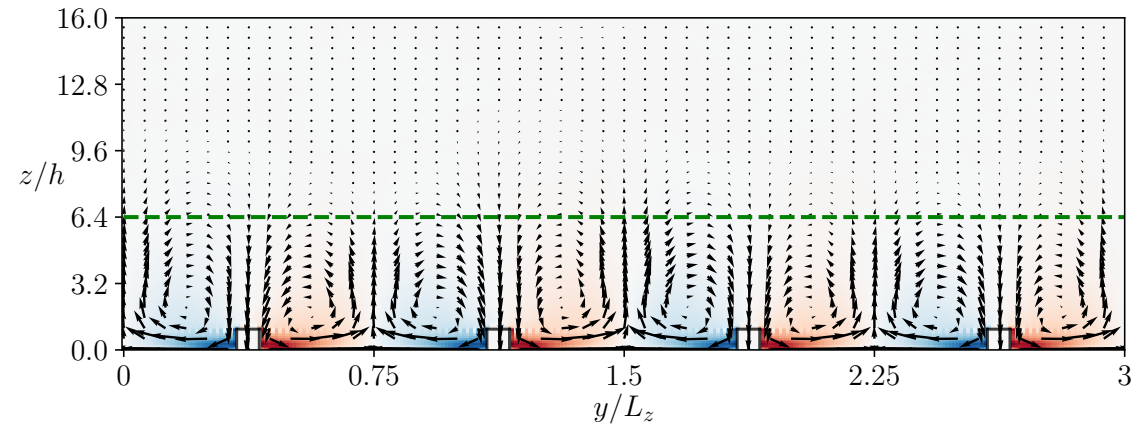
Sparse, $\lambda = 0.007$



Mean streamwise velocity profiles gradually diverge with decreasing packing density.

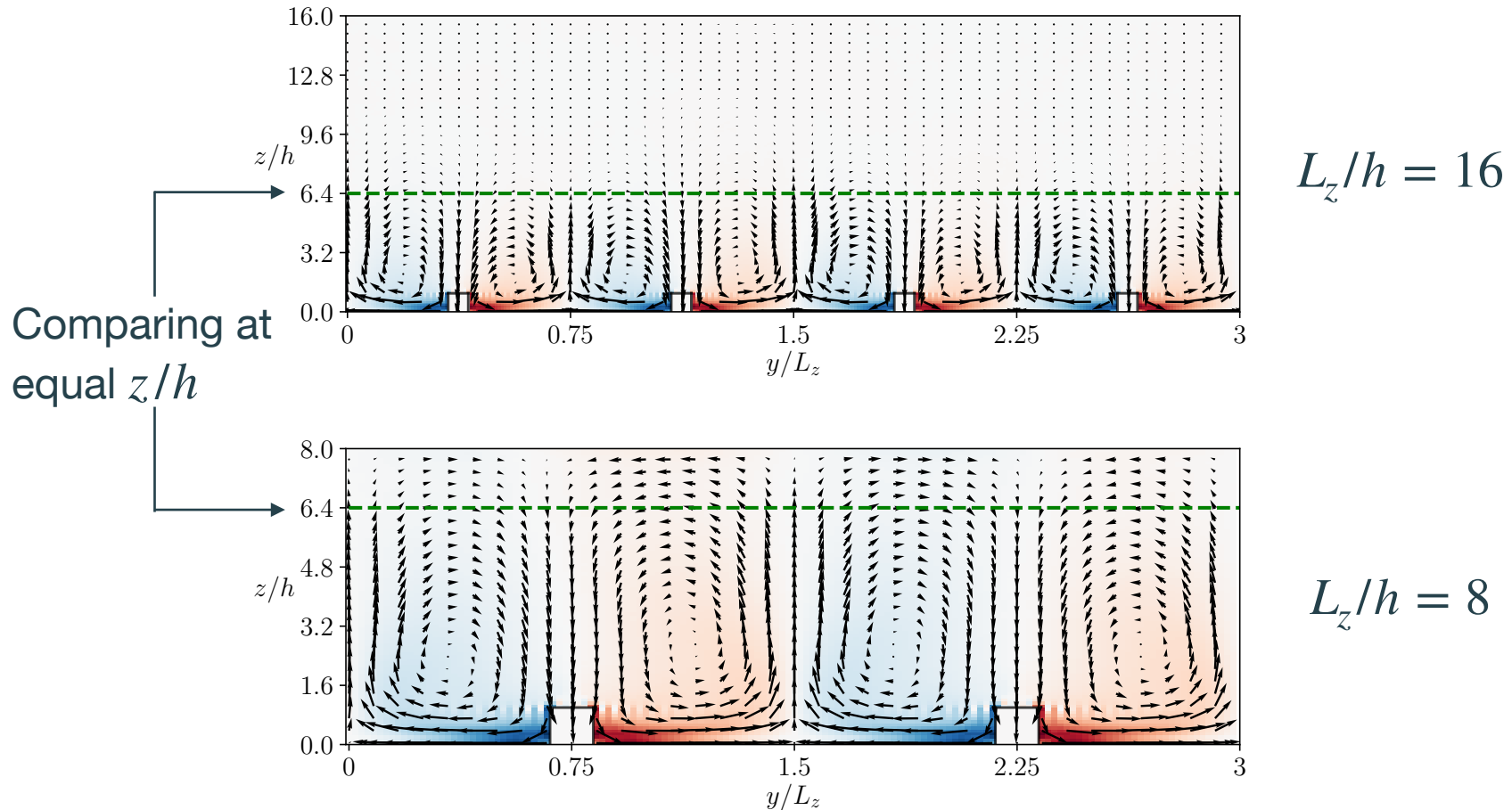


As we change the scale separation for sparse configuration, the size and strength of secondary flows changes.

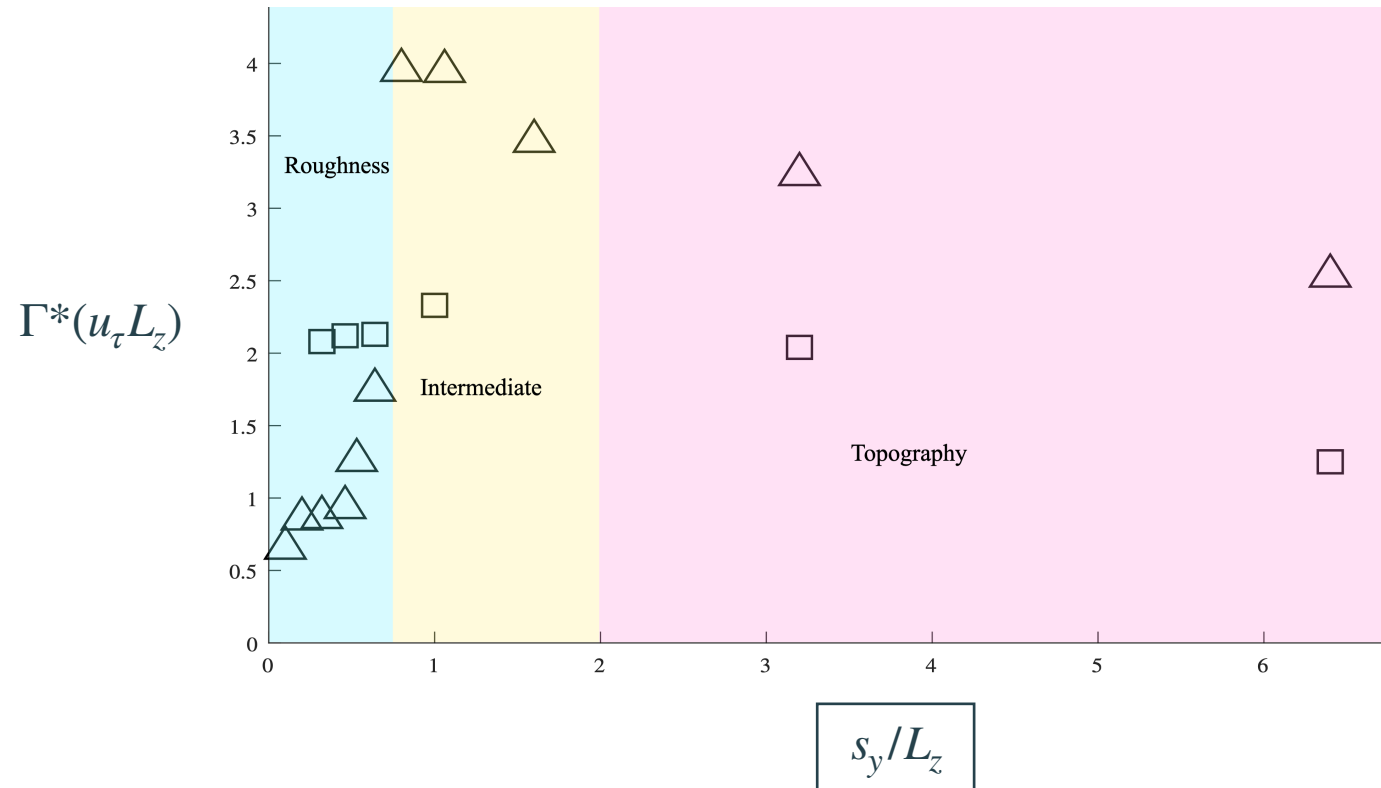


$$L_z/h = 16$$

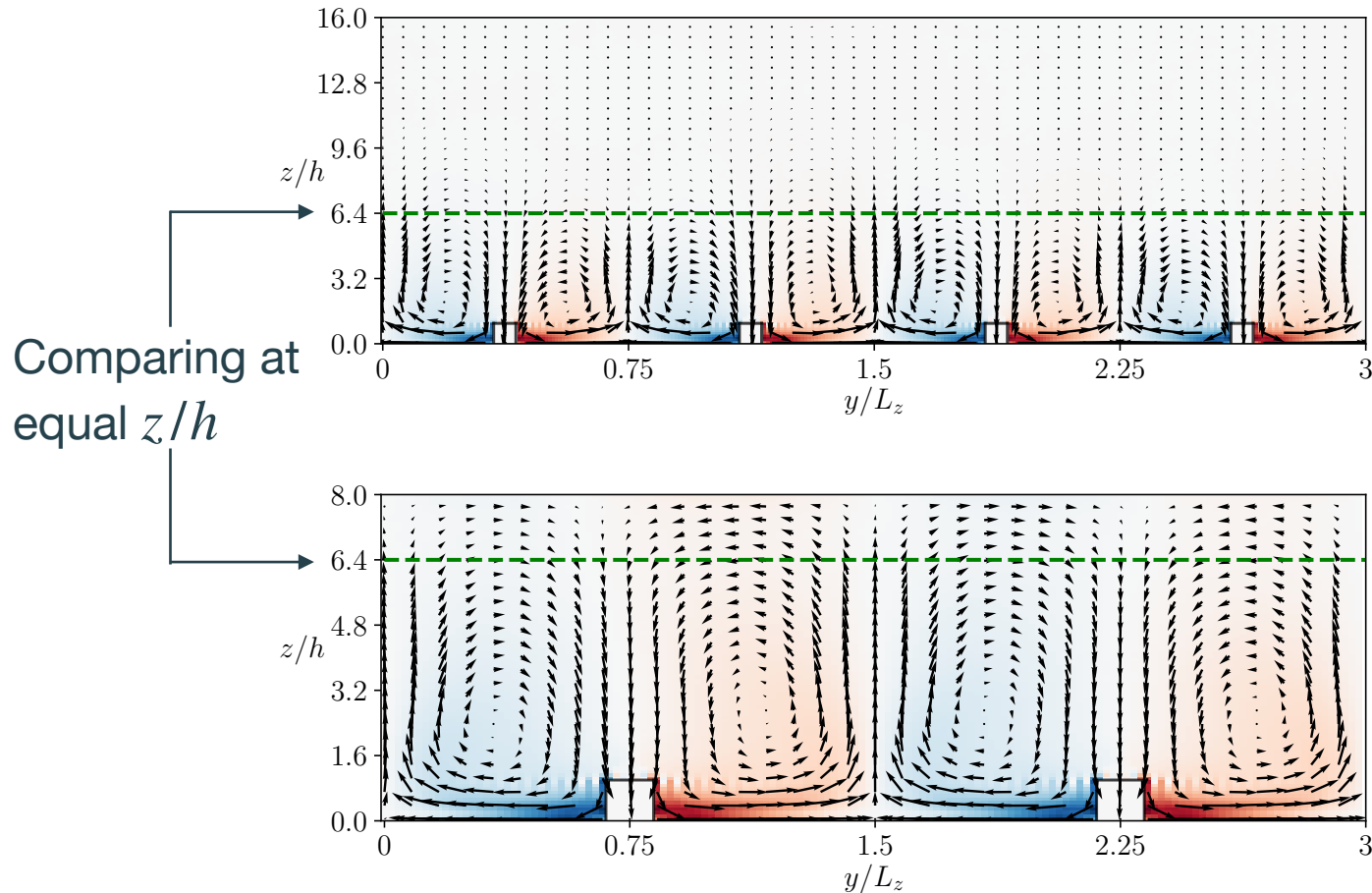
As we change the scale separation for sparse configuration, the size and strength of secondary flows changes.



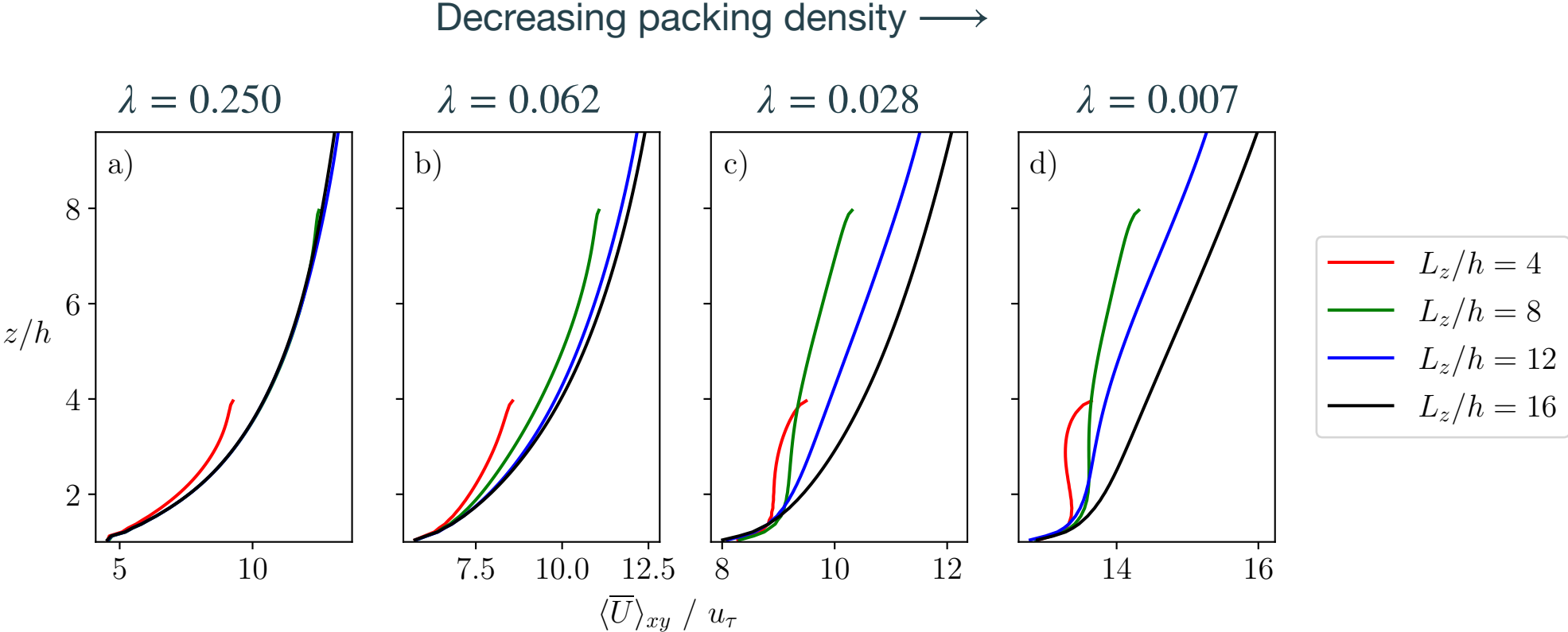
s_y/L_z is a crucial parameter that controls size and strength of secondary flows [Yang and Anderson (2017)].



When L_z/h changes from 16 to 8, s_y/L_z changes from 0.75 to 1.5.



The deviation in mean streamwise velocity for sparse configuration is not solely due to the impact of scale separation.



To analyze the individual impact of a single parameter, we systematically adjust that particular variable while maintaining constancy in all other parameters.

$$U_1 = f(a_1, b, c, d, e, \dots)$$



$$U_2 = f(a_2, b, c, d, e, \dots)$$



Impact of parameter a on U is isolated.

To analyze the individual impact of a single parameter, we systematically adjust that particular variable while maintaining constancy in all other parameters.

$$U_1 = f(a_1, b, c, d, e, \dots)$$



$$U_2 = f(a_2, b, c, d, e, \dots)$$



Impact of parameter a on U is isolated.

$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{h}, \frac{h_x}{h}, \frac{s_y}{h}, \frac{s_x}{h}, \frac{z}{h}\right)$$



Impact of parameter L_z/h on U/u_τ is isolated.

L : Domain length

h : Cuboid dimensions

s : Spacing between cuboids

To analyze the individual impact of a single parameter, we systematically adjust that particular variable while maintaining constancy in all other parameters.

$$U_1 = f(a_1, b, c, d, e, \dots)$$



$$U_2 = f(a_2, b, c, d, e, \dots)$$

← Impact of parameter a on U is isolated.

$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{h}, \frac{h_x}{h}, \frac{s_y}{h}, \frac{s_x}{h}, \frac{z}{h}\right)$$

← Impact of parameter L_z/h on U/u_τ is isolated.

- L : Domain length
- h : Cuboid dimensions
- s : Spacing between cuboids

When only L_z/h is varied while keeping s_y/h constant, it changes the value of s_y/L_z .

This limitation can be solved by rearranging the PI groups.

$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{h}, \frac{h_x}{h}, \frac{s_y}{h}, \frac{s_x}{h}, \frac{z}{h}\right)$$



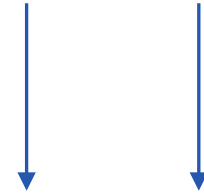
$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{h}, \frac{h_x}{h}, \frac{s_y}{L_z}, \frac{s_x}{h}, \frac{z}{h}\right)$$



s_y is adjusted when L_z/h varies, preserving the size and strength of secondary flows

This limitation can be solved by rearranging the PI groups.

$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{h}, \frac{h_x}{h}, \frac{s_y}{h}, \frac{s_x}{h}, \frac{z}{h}\right)$$



$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{L_z}, \frac{h_x}{h}, \frac{s_y}{L_z}, \frac{s_x}{h}, \frac{z}{h}\right)$$

[Willingham et al. (2014)]
[Hwang and Lee (2018)]

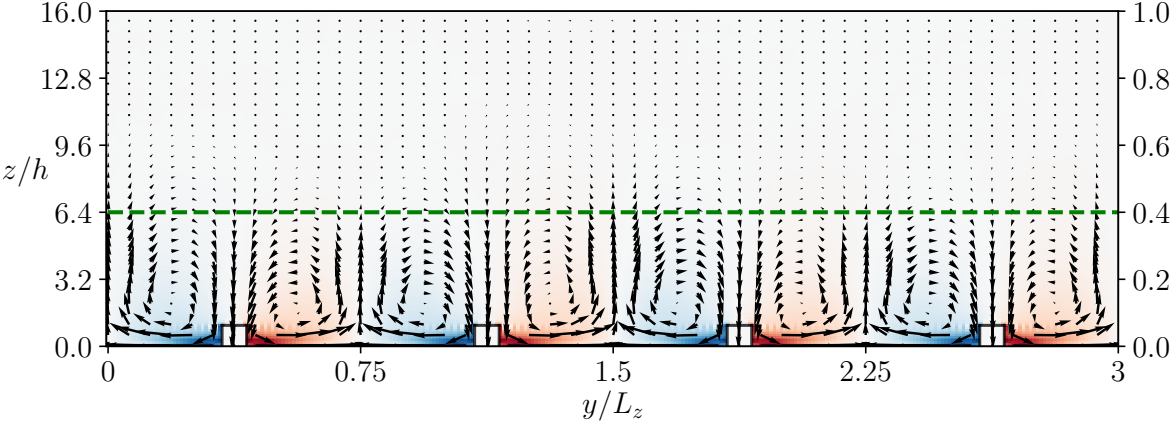
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↓ ↓ ↓

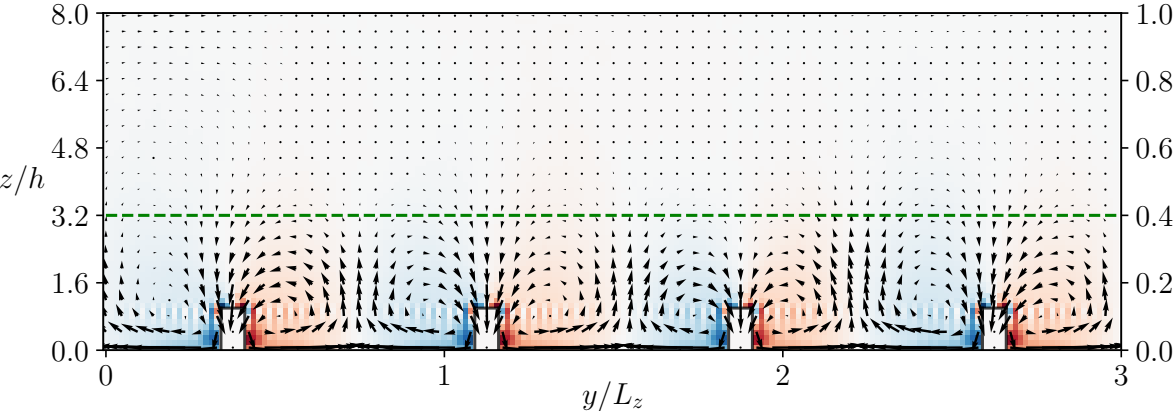
$$U/u_\tau = f\left(\frac{L_z}{h}, \frac{L_y}{L_z}, \frac{L_x}{L_z}, \frac{h_y}{L_z}, \frac{h_x}{h}, \frac{s_y}{L_z}, \frac{s_x}{h}, \frac{z}{L_z}\right)$$

With the new set of Pi groups, s_y/L_z is preserved across the simulations, generating equivalent secondary flow configurations.



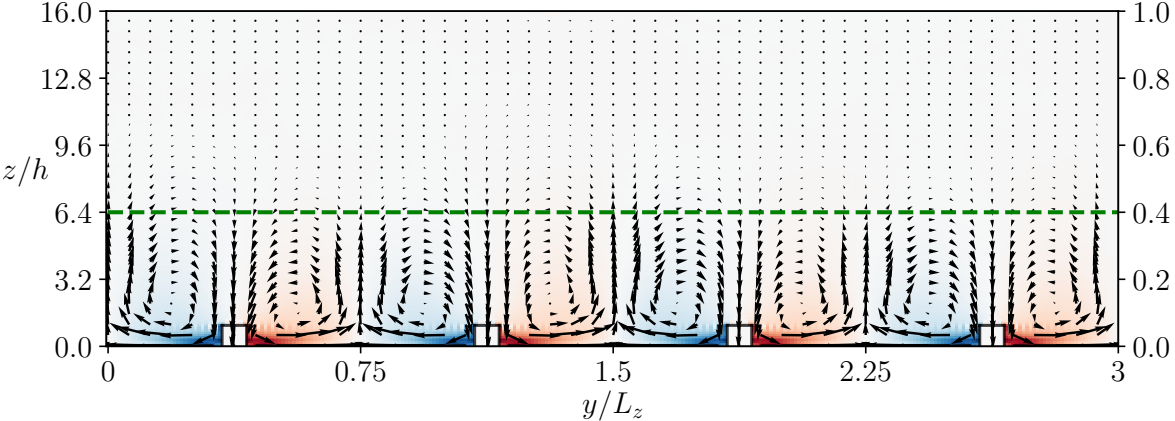
$L_z/h = 16, s_y/L_z = 0.75$

Comparing at
equal z/L_z



$L_z/h = 8, s_y/L_z = 0.75$

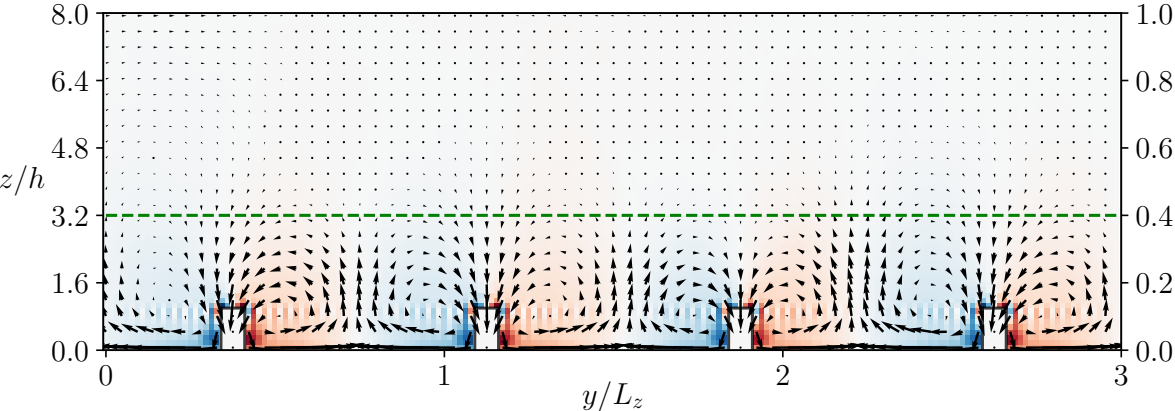
With the new set of Pi groups, s_y/L_z is preserved across the simulations, generating equivalent secondary flow configurations.



$$L_z/h = 16, s_y/L_z = 0.75$$

$$h_y/L_z = 1/16 \longrightarrow h_y = 1$$

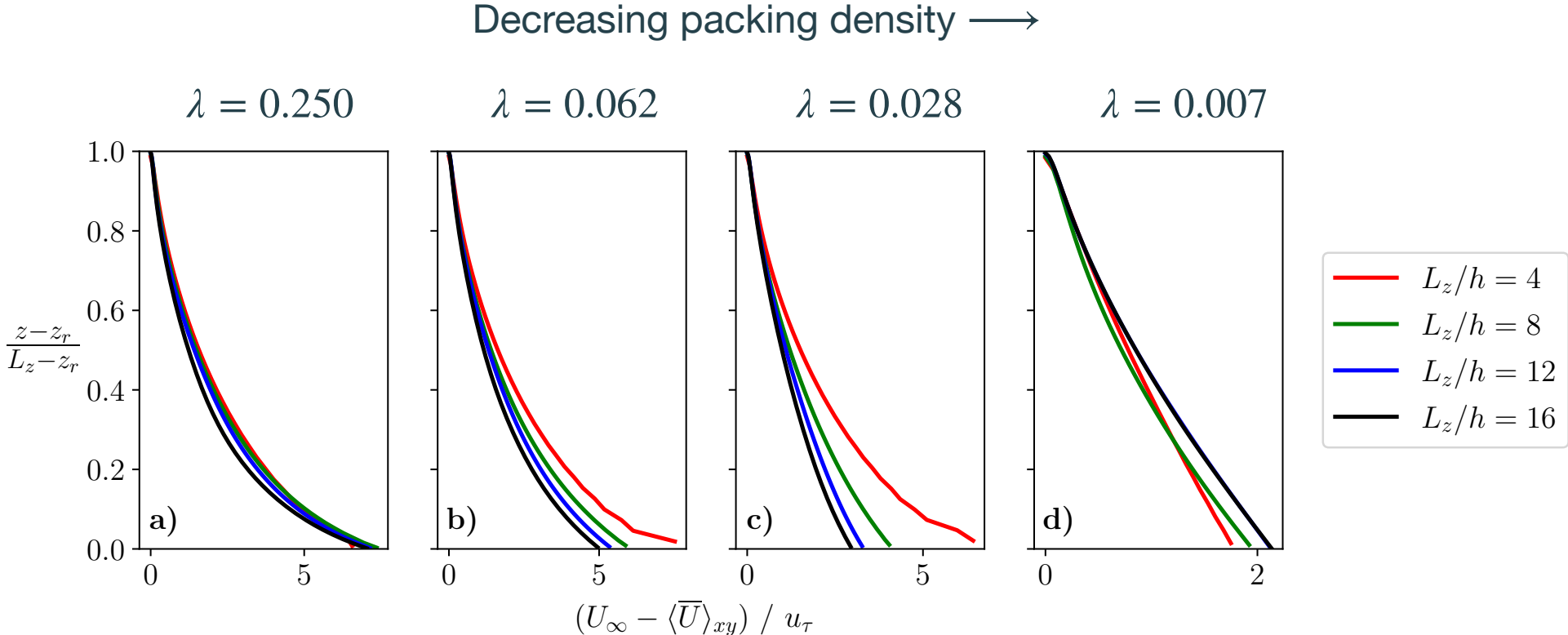
Comparing at
equal z/L_z



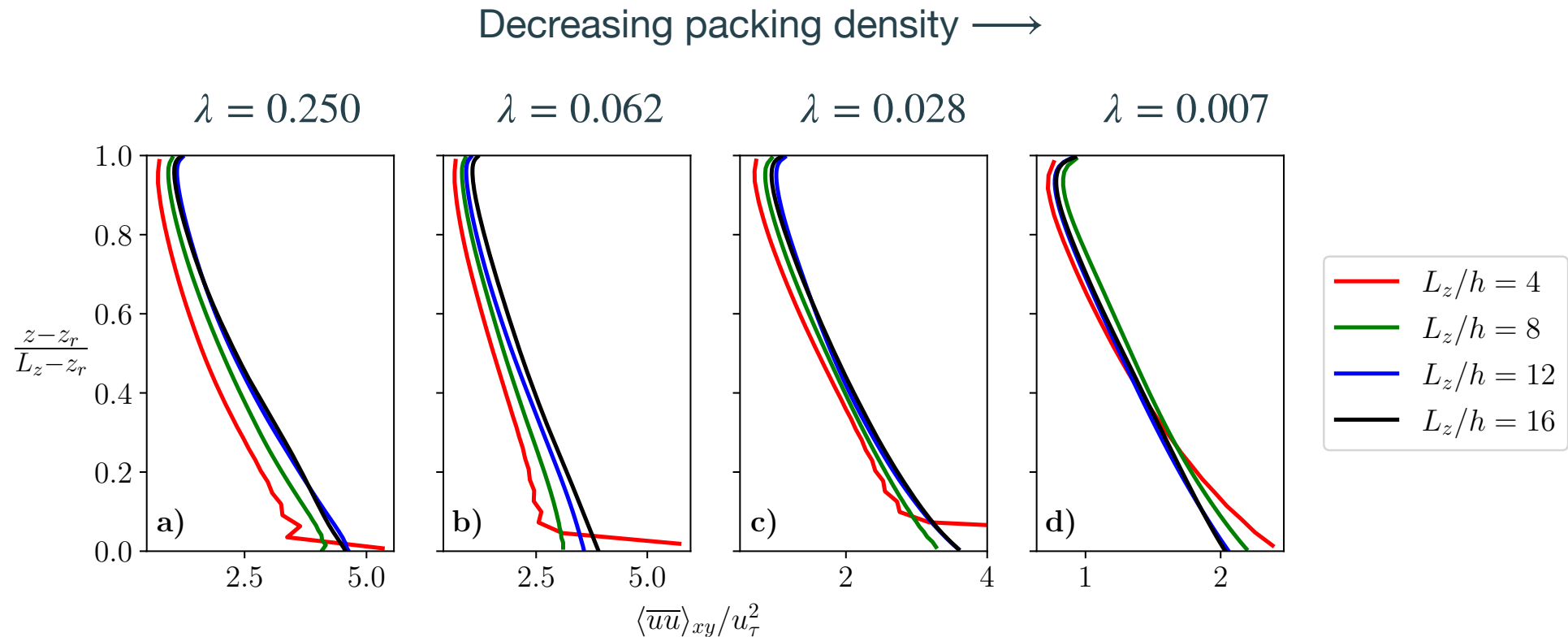
$$L_z/h = 8, s_y/L_z = 0.75$$

$$h_y/L_z = 1/16 \longrightarrow h_y = 0.5$$

Converging trend is observed with L_z based scaling across all the packing densities.



Converging trend is also observed for the 2nd order statistics.



In summary, conventional method to test the impact of scale separation only works for dense configurations.

A novel approach is shown to test the impact of scale separation, which requires producing equivalent surface geometry.

With this approach, we see that the scale separation of 12 - 16 is enough for most of the applications to minimize the artificial impact of top boundary condition.

Acknowledgements



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Advancing
Innovation



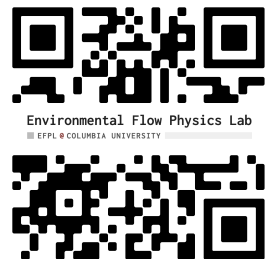
Contributions

Sathe, A., Giometto, M. G. (2023). Impact of the numerical domain on turbulent flow statistics; scalings and considerations for canopy flow. *Journal of Fluid Mechanics* (*under review*)

Sathe, A., Giometto, M. G. (2022), Impact of Numerical Domain on Turbulent Flow Statistics: Scalings and Considerations for Canopy Flows [Conference presentation], AGU fall meeting, Chicago, IL, United States.

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- Willingham, D., Anderson, W., Christensen, K. T., & Barros, J. M. (2014). Turbulent boundary layer flow over transverse aerodynamic roughness transitions: Induced mixing and flow characterization. *Physics of Fluids*, 26(2), 025111. <https://doi.org/10.1063/1.4864105>
- Hwang, H. G., & Lee, J. H. (2018). Secondary flows in turbulent boundary layers over longitudinal surface roughness. *Physical Review Fluids*, 3(1). <https://doi.org/10.1103/PhysRevFluids.3.014608>