Angular Dichroism in (e, 2e) Electron Scattering Processes

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A R T I C L E I N F O

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A B S T R A C T

The purpose of this research project was to investigate hydrogen in its ground (1S) and excited (3S) states undergoing ionization as a result of being impacted by electrons and provide a comparison of the DDCS for the two states. Through the application of multiple scattering theory to compute the DDCS, an emphasis has been placed on the influence of the initial quantum state upon the cross section for ionization and the angular distribution of the outgoing electrons. A comprehensive multiple scattering approach has been utilized to model the interaction between the incident electron and hydrogen atom, including elastic and inelastic scattering processes. Results obtained from our study have demonstrated significant differences in the threshold energy for ionization and the angular scattering patterns between the ground and excited states of hydrogen. These results illustrate the unique nature of the scattering behavior associated with the hydrogen atom in each electronic state and indicate the critical influence of the electronic state of the hydrogen atom upon the ionization process. Consequently, this research has substantially advanced the theoretical comprehension of ionization in atomic systems and offers a more precise model that can enhance theoretical frameworks and experimental data in atomic and plasma physics, particularly regarding low-energy electron-atom collisions.

Introduction

The investigation of atomic ionization using charged particle projectiles, such as electrons, is a main topic in atomic physics. Finding the specific numerical values of each integer representing the various cross-sections, that is., single, double [1], [2], and triple differential [3] on different representations of kinematics, is an exciting and fascinating area of applied mathematics. For the last 50 years, the field has benefited greatly from the results of experiments carried out in astrophysics, plasma physics, and fusion technology [4]-[18]. This paper examines the ionization of hydrogen atoms in a metastable state. We anticipate providing experimental results in due course. Our calculation method is based on the Lewis integral [19].

The double differential cross-section (DDCS) describes the angular and energetic distribution of secondary electrons created during atomic ionization collisions. DDCS information is significant in analyzing astrophysical and upper atmosphere phenomena and electron-impact spectra, as well as the secondary consequences of slow secondary electrons and the events studied by Das et al. [1]. Shyn [2] used differential double coincidence spectroscopy (DDCS) to evaluate the existing experimental data for the He atom and the distribution of angle and energy, which is probably the easiest experimentally. Bethe [3] is responsible for being the first to apply quantum mechanics to understand the ionization process of fast particles without utilizing relativistic corrections.

A multi-scattering theory [3] was used to compute the differential double cross section (DDCS) in order to examine the ionization of metastable 3S-state hydrogen atoms using electron impact at energies of 150 eV and 250 eV. The multi-scattering wave function [3], [4] was developed for a system containing two electrons in a Coulomb field and includes higher-order and correlation effects. Time-dependent cross sections (TDCS) were computed accurately with these wave functions, separately and in many kinematic conditions in electron-hydrogen ionization collisions for the ground and metastable 3S-states at non-relativistic energy levels [3], [4]. Das et al. [3] introduced a modest but important study of the current DDCS potential ionization of the metastable 3S-state of the hydrogen atom from electrons of intermediate energy.

The differential double cross section (DDCS) was expressed as an integral of the total double cross-section (TDCS) [9], [10], [11], and [12] over a range of electron directions, and then it was compared with the predictions of Shyn [2] and Das et al. [1]. They determined the DDCS values, and compared their results with those reported by Shyn [2] and Das et al. [1]. The wave function proposed by Das et al. [3], [4] is useful and interesting for the present study on metastable 3S-state hydrogen atom ionization by electrons.

Theoretical Method

Ionization cross-sections are based on the number of ionizations per unit time and per unit target to the incident electron flux. At the moment, the deepest knowledge is for single ionization processes of the type:

$$e^- + H(3S) \rightarrow H^+ + 2e^- \quad \textit{for electron impact} \qquad (1)$$

The notation in the preceding equation has been described in the caption and can be determined in coplanar geometry via the evaluation of a triple differential cross section (TDCS) measured in (e, 2e) coincidence experiments. By considering ionization of a hydrogen atom by electrons [3], the T-matrix can be expressed [4] by the following expression:

$$T_{FI}=\langle\Psi_F^{(-)}(\bar{\gamma}_a,\bar{\gamma}_b)|V_I(\bar{\gamma}_a,\bar{\gamma}_b)|\Phi_I(\bar{\gamma}_a,\bar{\gamma}_b)\rangle \qquad (2)$$

Here the perturbation potential $V_I(\gamma_a,\gamma_b)$ is given by

$$V_I(\bar{\gamma}_a,\bar{\gamma}_b)=\frac{Z}{\gamma_b}-\frac{Z}{\gamma_{ab}} \qquad (3)$$

Where "Z= -1 for electrons" is the nuclear charge of the hydrogen atom, $\bar{\gamma}_a$ and $\bar{\gamma}_b$ are the distances of the two electrons from the nucleus, and γ_{ab} is the distance between the two electrons.

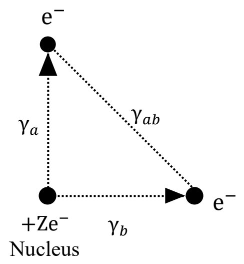

Figure a: Collision effect amongst two electrons and the nucleus.

The initial unperturbed channel wave function is given by:

$$\Phi_I(\bar{\gamma}_a,\bar{\gamma}_b)=\frac{e^{i\cdot\bar{p}_i\cdot\bar{\gamma}_b}}{(2\pi)^{\frac{3}{2}}}\phi_{3S}(\bar{\gamma}_a)=\frac{e^{i\cdot\bar{p}_i\cdot\bar{\gamma}_b}}{(2\pi)^{\frac{3}{2}}}\cdot\frac{1}{81\sqrt{3\pi}}\left(27\text{-}18\gamma_a+2\gamma_a^2\right)e^{-\lambda_a\gamma_a} \qquad (4)$$

Here

$$\phi_{3S}(\bar{\gamma}_a)=\frac{1}{81\sqrt{3\pi}}\left(27\text{-}18\gamma_a+2\gamma_a^2\right)e^{-\lambda_a\gamma_a} \qquad (5)$$

Here $\lambda_a=\frac{1}{3}$, $\phi_{3S}(\bar{\gamma}_a)$ is the hydrogen 3S-state wave function, and $\Psi_F^{(-)}(\bar{\gamma}_a,\bar{\gamma}_b)$ is the final three-particle scattering state wave function [3] with the electrons being in the continuum with momenta $\bar{p}_a\ and\,\bar{p}_b$. And The coordinates of the two electrons are $\bar{\gamma}_a\ and\ \bar{\gamma}_b$. Here the approximate wave function $\Psi_F^{(-)}(\bar{\gamma}_a,\bar{\gamma}_b)$ [4] is given by

$$\Psi_F^{(-)}(\bar{\gamma}_a,\bar{\gamma}_b)=\frac{N(\bar{p}_a,\bar{p}_b)\left[\phi_{\bar{p}_a}^{(-)}(\bar{\gamma}_a)e^{i\bar{p}_b\cdot\bar{\gamma}_b}+\phi_{\bar{p}_2}^{(-)}(\bar{\gamma}_b)e^{i\bar{p}_a\cdot\bar{\gamma}_a}+\phi_{\bar{p}}^{(-)}(\bar{\gamma})e^{i\bar{P}.\bar{R}}\text{-}2e^{i\bar{p}_a\cdot\bar{\gamma}_a+i\bar{p}_b\cdot\bar{\gamma}_b}\right]}{(2\pi)^3} \qquad (6)$$

Here $\bar{\gamma}=\frac{\bar{\gamma}_b-\bar{\gamma}_a}{2},\ \bar{R}=\frac{\bar{\gamma}_b+\bar{\gamma}_a}{2},\ \bar{p}=(\bar{p}_b\text{-}\bar{p}_a),\ \bar{P}=(\bar{p}_b+\bar{p}_a)$

The scattering amplitude [4] may be written as

$$F(\bar{p}_a,\bar{p}_b)=N(\bar{p}_a,\bar{p}_b)[F_{eT}+F_{PT}+F_{Pe}-2F_{PWB}] \qquad (7)$$

Where $F_{eT}, F_{PT}, F_{Pe}\ and\ F_{PWB}$ are the amplitudes corresponding to the four terms of Eq. (7) respectfully. Here $N(\bar{p}_a,\bar{p}_b)$ is the normalization constant, given by,

$$|N(\bar{p}_a,\bar{p}_b)|^{-2}=\left|7\text{-}2[\lambda_a+\lambda_b+\lambda_c]\text{-}\left[\frac{2}{\lambda_a}+\frac{2}{\lambda_b}+\frac{2}{\lambda_c}\right]+\left[\frac{\lambda_a}{\lambda_b}+\frac{\lambda_a}{\lambda_c}+\frac{\lambda_b}{\lambda_a}+\frac{\lambda_b}{\lambda_c}+\frac{\lambda_c}{\lambda_a}+\frac{\lambda_c}{\lambda_b}\right]\right| \qquad (8)$$

Where

$$\lambda_a=e^{\frac{\pi\alpha_a}{2}}\Gamma(1\text{-}i\alpha_a),\quad \alpha_a=\frac{1}{p_a};$$

$$\lambda_b=e^{\frac{\pi\alpha_b}{2}}\Gamma(1\text{-}i\alpha_b),\quad \alpha_b=\frac{1}{p_b};$$

$$\lambda_c=e^{\frac{\pi\alpha}{2}}\Gamma(1\text{-}i\alpha),\quad \alpha=-\frac{1}{p}.$$

Here $\phi_{\bar{q}}^{(-)}(\bar{\gamma})$ is the coulombs wave function, given by,

$$\phi_{\bar{q}}^{(-)}(\bar{\gamma})=e^{\frac{\pi\alpha}{2}}\Gamma(1+i\alpha)e^{i\bar{q}\cdot\bar{\gamma}}\ {}_1F_1(-i\alpha,1,-i[q\gamma+\bar{q}.\bar{\gamma}]) \tag{9}$$

The general one-dimensional integral representation of the confluent hyper geometric function is written by,

$$ {}_1F_1(a,c,z)=\frac{\Gamma(c)}{(a)\Gamma(c-a)}\int_0^1 dx\, x^{(a-1)}(1-x)^{(c-a-1)}e^{(xz)} \tag{10}$$

For the electron impact ionization, the parameters α_a, α_b *and* α are given by,

$$\alpha_a=\frac{1}{P_a}\ for\ \bar{q}=\bar{p}_a\ ,\ \alpha_b=\frac{1}{p_b}\ for\ \bar{q}=\bar{p}_b\ \ and\ \ \alpha=-\frac{1}{p}\ for\ \ \bar{q}=\bar{p}.$$

Now applying equations (5) and (7) to the equation (3), we get

$$\mathrm{T_{FI}}=\mathrm{N}(\bar{\mathrm{p}}_\mathrm{a},\bar{\mathrm{p}}_\mathrm{b})[\mathrm{T_B}+\mathrm{T_{B'}}+\mathrm{T_I}-2\mathrm{T_{PB}}] \tag{11}$$

Where

$$T_B=\langle\Phi_{\bar{p}_a}^{(-)}(\bar{\gamma}_a)\,e^{i\bar{p}_b\cdot\bar{\gamma}_b}|\,V_I\,|\,\Phi_I(\bar{\gamma}_a,\bar{\gamma}_b)\rangle \tag{12}$$

$$T_{B'}=\langle\Phi_{\bar{p}_b}^{(-)}(\bar{\gamma}_b)\,e^{i\bar{p}_a\cdot\bar{\gamma}_a}|\,V_I\,|\,\Phi_I(\bar{\gamma}_a,\bar{\gamma}_b)\rangle \tag{13}$$

$$T_I=\langle\Phi_{\bar{p}}^{(-)}(\bar{\gamma})\,e^{i.\bar{P}.\bar{R}}\,|\,V_I\,|\,\Phi_I(\bar{\gamma}_a,\bar{\gamma}_b)\rangle \tag{14}$$

$$T_{PB}=\langle e^{i\bar{p}_a\cdot\bar{\gamma}_a+i\bar{p}_b\cdot\bar{\gamma}_b}\,|\,V_I\,|\,\Phi_I(\bar{\gamma}_a,\bar{\gamma}_b)\rangle \tag{15}$$

For the first-born approximation, equation (12) may be written as

$$T_B=\frac{1}{162\sqrt{6}\pi^2}\langle\Phi_{\bar{p}_a}^{(-)}(\bar{\gamma}_a)\,e^{i.\bar{p}_b\cdot\bar{\gamma}_b}\left|\frac{1}{\gamma_{ab}}-\frac{1}{\gamma_b}\right|e^{i.\bar{p}_i\cdot\bar{\gamma}_b}\,(27-18\gamma_a+2\gamma_a^2)\,e^{-\lambda_a\cdot\gamma_a}\rangle$$

From the equation (13), For the second-born approximation may be written as,

$$\mathrm{T_{B'}}=\frac{1}{162\sqrt{6}\pi^2}\langle\Phi_{\bar{p}_b}^{(-)}(\bar{\gamma}_b)e^{i\bar{p}_a\cdot\bar{\gamma}_a}\left|\frac{1}{\gamma_{ab}}-\frac{1}{\gamma_b}\right|\Phi_i(\bar{\gamma}_a,\bar{\gamma}_b)\rangle$$

$$=\frac{1}{162\sqrt{6}\pi^2}\int\Phi_{\bar{p}_b}^{(-)*}(\bar{\gamma}_b)e^{-i\bar{p}_a\cdot\bar{\gamma}_a}\left|\frac{1}{\gamma_{ab}}-\frac{1}{\gamma_b}\right|e^{i\bar{p}_i\cdot\bar{\gamma}_b}(27-18\gamma_a+2\gamma_a^2)e^{-\lambda_a\gamma_a}d^3\gamma_a d^3\gamma_b$$

The third term of equation (14) can be written as follows:

$$\mathrm{T_I}=\frac{1}{162\sqrt{6}\pi^2}\langle\Phi_{\bar{p}}^{(-)*}(\bar{\gamma})e^{i\bar{P}.\bar{R}}\left|\frac{1}{\gamma_{ab}}-\frac{1}{\gamma_b}\right|(27-18\gamma_a+2\gamma_a^2)e^{i\bar{p}_i\cdot\bar{\gamma}_b}e^{-\lambda_a\gamma_a}\rangle$$

$$\mathrm{T_I}=\frac{1}{162\sqrt{6}\pi^2}\int\Phi_{\bar{p}}^{(-)*}(\bar{\gamma})e^{i\bar{P}.\bar{R}}\left|\frac{1}{\gamma_{ab}}-\frac{1}{\gamma_b}\right|(27-18\gamma_a+2\gamma_a^2)e^{i\bar{p}_i\cdot\bar{\gamma}_b}e^{-\lambda_a\gamma_a}\,d^3\gamma_a d^3\gamma_b$$

The last term of equation (15) can be written as follows:

$$\mathrm{T_{PB}}=\frac{1}{162\sqrt{6}\pi^2}\langle e^{-i\bar{p}_a\cdot\bar{\gamma}_a}e^{-i\bar{p}_b\cdot\bar{\gamma}_b}\left|\frac{1}{\gamma_{ab}}-\frac{Z}{\gamma_b}\right|e^{i.\bar{p}_i\cdot\bar{\gamma}_b}(27-18\gamma_a+2\gamma_a^2)e^{-\lambda_a\cdot\gamma_a}\rangle$$

$$=\frac{1}{162\sqrt{6}\pi^2}\int e^{-i\bar{p}_a\cdot\bar{\gamma}_a}e^{-i\bar{p}_b\cdot\bar{\gamma}_b}\left|\frac{1}{\gamma_{ab}}-\frac{1}{\gamma_b}\right|(27-18\gamma_a+2\gamma_a^2)e^{i\bar{p}_i\cdot\bar{\gamma}_b}\,e^{-\lambda_a\cdot\gamma_a}d^3\gamma_a d^3\gamma_b$$

The above equation is also analytically derived in our present study, whereby we used the Lewis Integral [19]. In addition, the TDCS we acquire corresponds to the T-matrix and is given by:

$$\frac{d^3\sigma}{dE_a d\Omega_a\,d\Omega_b}=\frac{p_a\,p_b}{p_i}|T_{FI}|^2 \tag{16}$$

And the DDCS corresponding to Eq. (16) can be obtained through the integration [8] with respect to solid angle Ω_b.

$$\frac{d^2\sigma}{dE_a\,d\Omega_a}=\int\frac{d^3\sigma}{dE_a\,d\Omega_a\,d\Omega_b}d\Omega_b \tag{17}$$

Hence, the result was generated by the numerical calculation using a programming language, MATLAB.

Results and Discussions

We presented the double differential cross sections (DDCS) for ionizing the hydrogen target in the 3S state due to electron impact, as a function of ejected electron energies E_a at incident energies E_I of 150 eV and 250 eV. The plots are a function of the ejection angles θ_a(0°–360°) with fixed scattered electron angles (scattering angle θ_b from 0°–90°) perspective. We only show the main figures involving θ_a(0°–60°) for recoil and binary cases. We compare the present DDCS to the experimental data for ground state (1), first-born data Shyn [2] and the theoretical results of Das et al. [1]; we also plot first-born for your convenience. The recoil region is θ_a = 0°–90° at ϕ = 0° and the binary region is θ_a= 90°–180° at ϕ= 180°.

Figure 1. Double Differential Cross Section (DDCS) for incident energy (E_I=250eV) and energy transfer (E_a=4eV) We see qualitatively a forward (or hit as peak) and slightly concerning event at larger θ_a. Qualitatively, this matches Shyn [2] experimental results; at present, DDCS is a qualitative case. The ground state results of Shyn [2] matched the records of Das et al. [1] in the recoiled case. The firstborn also experiences a presumably similar event, but in qualitatively different circumstances. The electron differential double cross

section, DDCS, has two peaks that celeripede to the recoil and binary lobes, and the electron results are inferior to the electron DDCS in the recoil area of about $\theta_{a}(0° - 100°)$ and the binary lobes of $\theta_{a}(90° - 160°)$.

Figure 2. shows the Double Differential Cross Section, DDCS, for an incident energy, $E_I = 250$eV and an energy transfer, $E_a = 10$eV. The present 3S calculation and results of Das et al. (1995) qualitatively match and will likely be in reasonable agreement, especially in the binary region. At larger angles, the results of Shyn better agree with the present calculations than those of Das et al. In addition, the present DDCS is slightly larger than both Das and Shyn in the binary region, and in the charge transfer region the present calculation matches both references. The current second-born DDCS and first-born DDCS are virtually indistinguishable in this kinematic analysis.

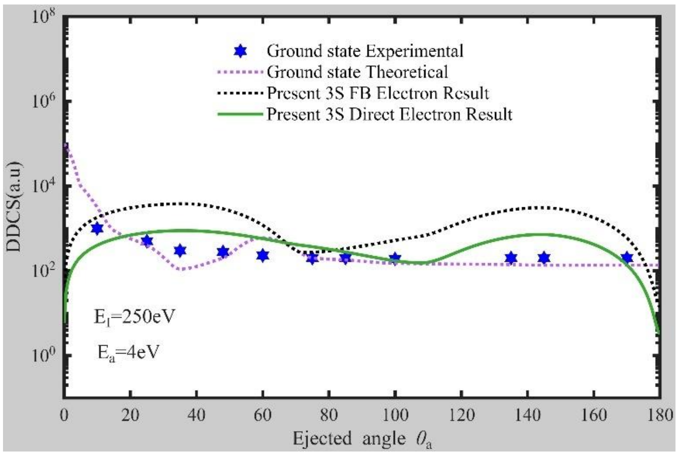

Figure 1: Second Born double differential cross section (DDCS) as a function of electron impact energy $E_I = 250\text{eV}$ and ejected electron energy $E_a = 4\text{eV}$.

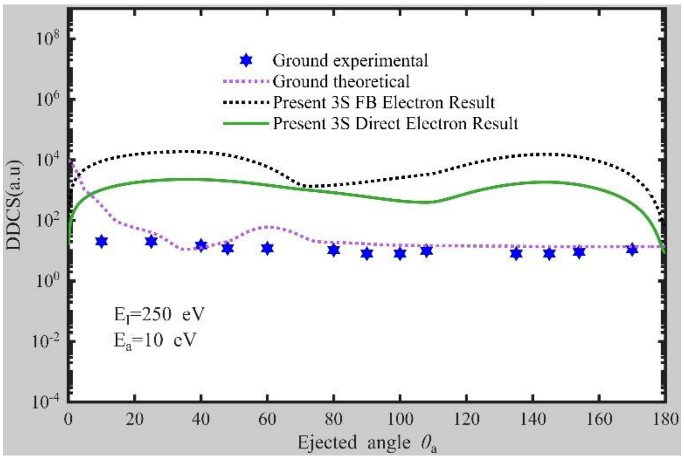

Figure 2: Second Born double differential cross section (DDCS) for electron impact energy $E_I = 250\text{eV}$ and ejected electron energy $E_a = 10\text{eV}$.

Additionally, we illustrate the initial results of the incident electron energy and the ejected electron energy in Figure 3; the two peaks corresponding to the recoil and binary regime structures are evident in the initial results, and the findings for high ejected energy are in strong agreement with the experimental results. The residual computations and the hydrogen state outcomes from Das et al. [1] are congruent. Conversely, other experimental data exhibit an opposing profile in the recoil region; nonetheless, the current results align effectively with Shyn's experimental findings [2] in the binary region, suggesting that the present computations hold greater significance under the kinematic conditions.

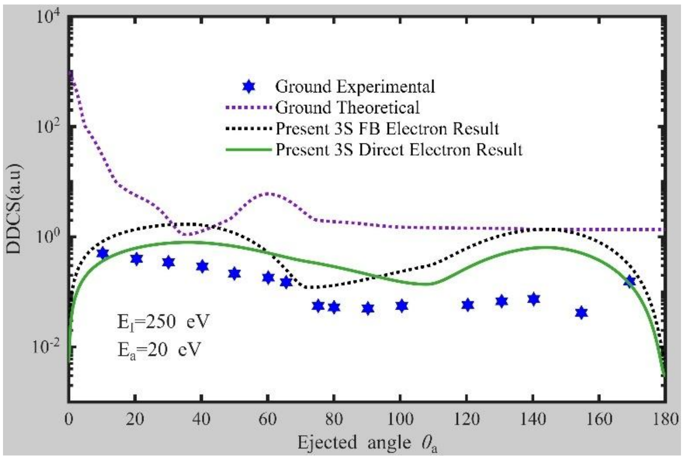

Figure 3: Second Born double differential cross section (DDCS) for the electron impact energy $E_I = 250\text{eV}$ and the ejected electron energy $E_a = 20\text{eV}$.

The findings indicate an excitation energy of 250eV, aligning well with all differential cross-section measurements and studies. Furthermore, all computed DDCS results for the metastable 3S-state closely align with Shyn's [2] experimental findings in the binary zone, particularly evident in this kinematic context. The hydrogen second Born result exhibits notable discrepancies when juxtaposed with the findings for the hydrogen ground state [1], [2]. This outcome pertains to the kinematics of two-body electron-electron interactions and the final state interaction between the electron and nucleus, resulting in a broad peak-shaped pattern in the double differential cross sections for the ejected electron at an incident energy of 250eV following electron impact. In summary, this strategy has yielded results that are qualitatively comparable to prior findings on a hydrogen ground state.

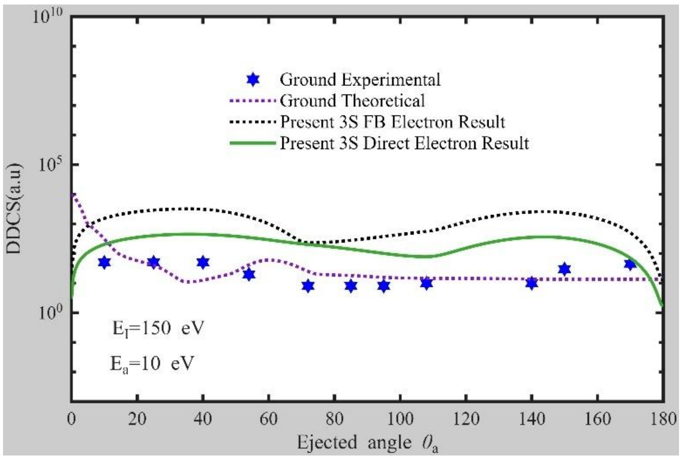

Figure 4: The Second Born double differential cross section (DDCS) for electron impact energy $E_I = 150eV$ and ejected electron energy $E_a = 10eV$.

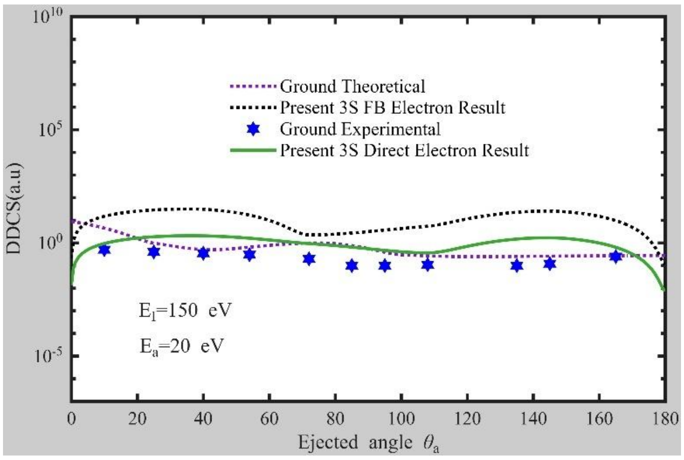

Figure 5: Second Born double differential cross section (DDCS) for electron impact energy $E_I = 150eV$ and ejected electron energy $E_1 = 20eV$.

Final Assessment This section assesses E_I=150 eV incident energy for ejected energies of E_a= 10eV, 20eV, and 50eV and is presented in Figure 4, Figure 5, and Figure 6, respectively.

Initially, in Figure 5, our results for the incident energy E_I=150eV and E_a=20eV qualitatively match the hydrogen ground state [3] and [4] results since we are above 50° on the ejection angle. Next, it goes above both comparisons and presents a smooth peak in a binary logic sense. First-born data sets also follow the same overall pattern, with the distinction of dips observed in the recoil region. In any case of electron impact at especially lower energies, our current and first-born results match the hydrogen ground state [3], and [4] results both below and above at binary angles.

As to the current recoil, we show the smooth bump, and it is not visible in the current but rather more in the current. In the binary peak area, it seems to slow down at an ejected value of E_I=10eV, and an impact value of E_I=150eV in Figure 6, in the recoil region, along with Shyn [4], at a greater ejected angle, and hydrogen ground state [3] and [4]. The calculations are very close to matching in the first result, and they show a strong relation with all measurements and angles respectively, and the recoil lobe is corroborated from in the angular distributions. The DDCS's first and second born approximately match the reference curves, and the end results have the same behavior compared to the kinematic states, with qualitative justification between theoretical predictions and experimental results for comparison.

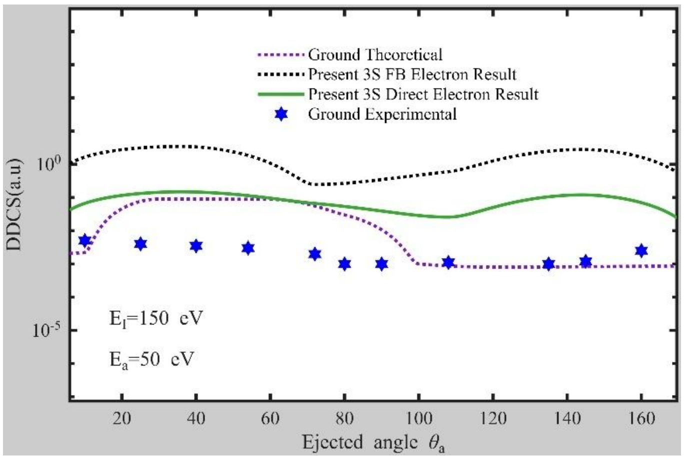

Figure 6: Second Born double differential cross section (DDCS) for electron impact energy $E_I = 150eV$ and ejected electron energy $E_a = 50eV$.

To understand these structures of the DDCS results one may look carefully to the Table-1.

θ_b (deg)	θ_a (deg)	DDCS E_a=10eV	DDCS E_a=20eV	DDCS E_a=50eV
0	0	0.0058E-03	0.0137E-04	0.0010
1	36	0.8886E-03	2.0982E-04	0.1487
2	72	0.3972E-03	0.9378E-04	0.0665
4	108	0.1539 E-03	0.3634E-04	0.0257
10	144	0.7179E-03	1.6951E-04	0.1201
20	180	0.0032E-03	0.0076E-04	0.0005
30	216	1.0109E-03	2.3871E-04	0.1691
40	252	0.0161E-03	0.0380E-04	0.0027
60	288	0.2904E-03	0.6858E-04	0.0486
90	324	0.5271E-03	1.2446E-04	0.0882
100	360	0.2077E-03	0.4904E-04	0.0347

Table 1: DDCS results for ejected angles θ_a corresponding to various scattering angles θ_b for E_a=10eV, E_a=20eV, E_a=50eV in ionization of hydrogen atoms for $E_I = 150$eV electron.

Furthermore, they raised the emitted energy, thus E_a=50 eV, and the incident energy E_I=150 eV. Go back to the DDCS from the metastable 3S state, published by Das et al. [3] In the recoil region, it also is one region less than the recoil region of Shyn [2], and it is qualitatively contrasting in comparison to Shyn [2] for the same region, and it completely merges with Shyn [2] in the binary region. The firstborn estimate matched [1] and [2] for small-angle emission, and it currently matches Shyn [2] in the large-angle emission, and the recoil limit makes our results have even a satisfactory qualitative match to the hydrogen ground state results in [1] and [2].

However, there is a grand gap with larger initial values, and the first showed satisfactory updates to around five sig. figs., yet we do observe the variations associated with the number of states in hydrogen atomic states, and our current research seems to relate qualitatively to both hydrogen ground states suggested.

Conclusion

Although we have shown data for the double differential cross section (DDCS) for ionizing a metastable hydrogen atom (in the 3S state) by electron impact to demonstrate the validity of the theory of Das et al. [1], we have seen a large number of broad and smooth forward peaks, with larger emission angle values than previously found by Shyn [2], Bethe [3], and Das et al. [1]. However, the differences between the two types of hydrogen atoms were significantly larger when the hydrogen was in the ground state than when it was in the metastable state. Nevertheless, the second-born DDCS calculations made in this paper exhibit qualitative similarities to those calculated for hydrogen atoms in the ground state. These results represent theoretical advancements in the understanding of the electron ionization of hydrogen atoms in an excited state (i.e., the metastable 3S state).

As mentioned earlier, there are currently no experimental DDCS results available for comparison with the theoretical results of the ionization of the metastable 3S state; therefore, these results can serve as a theoretical benchmark for future experimental work. Further investigation could involve studying a broader range of incidents and ejected energies from various hydrogen metastable states, as well as other possible mechanisms of ionization due to electron impact. Additionally, noting that the current theoretical predictions extend the use of relativity and higher-order couplings and will help to provide further insight into our current predictions on the interaction of electrons with the atom during the initiation of the collision process in the excited state system.

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Conflicts Of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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