A Comparative Analysis of Theories of Quantum Gravity and Their Respective Approaches to the Information Paradox

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A thesis completed in partial fulfillment for the degree of
Master of Science in Physics

December 13, 2022

DOI: 10.13140/RG.2.2.22533.22241/8

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Acknowledgments

I can barely begin to express my gratitude to Dr. Timm Wrase for dedicating himself as my advisor and for his invaluable contributions of resources and feedback. Were it not for his decision to accept me under his wing, this endeavor would not have been possible. No less, his willingness to entertain my questions (no matter how inane), has made this process all the more rewarding and enjoyable.

I would also like to recognize Dr. Michael Stavola. As the chair of the Department of Physics at Lehigh University, it has been by his good graces that I have had the opportunity to serve as a teaching assistant during this master's program. The financial benefits from which have allowed me to better focus on my studies and this thesis.

I could not have undertaken this journey without the constant support from the multitude of excellent educators from whom I have had the pleasure of learning. In all levels of education, I have been beyond fortunate in having no shortage of such dedicated role models. I would like to extend a particular thanks to Dr. Sera Cremonini, Mr. Andrew Donnelly, Dr. Rhett Herman, Dr. Shawn Huston, Dr. Ariel Sommer, Dr. Brett Taylor, and Dr. Timm Wrase. While many other teachers and professors have served to guide me along this path, it is these who stand out most strongly in my mind as having made decisive contributions to bring me to where I am today.

My path through academia thus far has been anything but a straight line. Despite the struggles I have faced, it is through the compassion and aid from those like Dr. Irvin Clark, Dr. Rhett Herman, and Dr. Kathleen Hutnik that have allowed for my continued advancement in spite of these challenges.

Many thanks to my colleagues who have provided not only an atmosphere conducive to learning, but a sense of belonging and thereby made this process infinitely more pleasant. While I have pursued these studies out of a love for discovering our universe, it would be an unbearably lonely endeavor without all of you.

Special thanks to all my loved ones. It is through your constant support and encouragement that I have found the ability to pursue this passion of mine. It is impossible to overstate the significance of your role in affording me the opportunities I have had through your emotional, financial, and personal support. I would be remiss to not specifically name my mother, father, stepmother, and each of my grandparents for all the love they have expressed to me and the assistance they have provided. My mother, in particular, has offered more profound of support than I could have ever asked for despite her own immense challenges that she has faced during this time.

Lastly, I would like to recognize my cat, Bellatrix. Nobody else has sat with me for so long as I have prepared this thesis, nor brought me so much joy.

In deep appreciation for all who have helped me along this path - mentioned by name here or not:

Thank you.

Abstract

This paper provides descriptions of classical and quantum information to provide necessary background knowledge in the subsequent review of work pertaining to the development of the black hole information paradox. Said review includes discussions on the conservation of information, the information content of black holes (i.e. the extremal value of the Bekenstein bound), and the emergence of Hawking radiation from quantum fields in maximally curved spacetime. String Theory, Loop Quantum Gravity, Entropic Gravity, Causal Dynamical Triangulation, Hořava-Lifshitz Gravity, and Twistor Theory are explored with consideration for the current status of their development, what (if any) solutions they provide for the information paradox, and notable open problems within these theories. These various approaches to quantum gravity have been selected for the benefit of a variety of perspectives and on the subjective basis of personal interest - other approaches not discussed here may exist, and their exclusion from this thesis should not be construed as a disregard for the efforts contributed to their respective developments or any merit they may hold towards being accurate descriptors of gravitational phenomena.

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Part I

Introduction

The beginning of the 20^{th} century brought with it two revolutions in physics - the development of quantum mechanics and relativity. While each of these fields are highly successful in their own right, they conflict with each other at the high energy limit - producing conflicting or nonsensical results. While some success has been found through the use of perturbation theory in allowing quantum fields to be placed on curved spacetimes, there remain significant limits on such approaches. These difficulties in combining quantum mechanics with relativity has encouraged the development of numerous different approaches to better describe physical phenomena at a more fundamental level. Recent developments in astronomical observations (namely gravitational waves via interferometry and black hole imaging via the Event Horizon Telescope collaboration) have provided confirmation of the existence of black holes which previously only offered indirect evidence of their existence. With this confirmation, it is now clear that developing a theory to describe physics at such extreme limits has practical applications towards our understanding of the real universe rather than simply being of mathematical curiosity. Therefore, the black hole information paradox is a question of pivotal importance to the development of such a theory of quantum gravity. It roots from attempts to understand quantum field theory at a horizon such as one found under the extreme gravitational conditions of a black hole and results in questions as to if quantum determinism (wherein the time evolution operator determines the future state of a given wave function) and quantum reversibility (that the time evolution operator has an inverse) are valid principles on the Planck scale. Fundamentally, one may not have a complete theory of quantum gravity without a solution to this paradox. Numerous solutions have been postulated and various other theories of quantum gravity may yet still provide new potential solutions with time.

To simplify notation, dimensionless units have been used such that $c = \varepsilon_0 = \mu_0 = \hbar = G = k_B = 1$.

Part II

Foundations

0.1 Information

Information is structured and implicitly meaningful data which is inherent to any description of a physical system. It comes in the forms of classical information and quantum information, which are distinctly capable of describing physics at different scales.

0.1.1 Classical Information

Classical information is that which can be stored and transmitted as a string comprised of binary bits. This is information as laymen tend to understand it and applies to notions such as the information capacity of a computer hard drive or more generally any physics wherein $\frac{1}{p \cdot R} \approx 0$ (such that p is momentum and R is distance)[1]. Shannon entropy is the measure of classical information.

Bits: A bit, or "binary digit", is the representation of a Boolean state (1 or 0, true or false, etc.). A string of bits may collectively act to describe a physically meaningful system. A bit may be understood as a unit such a meter or second, albeit not an SI-defined unit.

Shannon Entropy: Shannon entropy is defined by:

$$H = -\sum_{i=1}^{n} \left(P_i \cdot \log_2 \left(P_i\right)\right) \tag{1}$$

Where n is the number of states, and P_i is the probability of each state appearing [1]. For example, given a bag with six marbles (three red, two blue, and one green), the Shannon entropy would be:

$$-\left(\frac{3}{6}\log_2\left(\frac{3}{6}\right) + \frac{2}{6}\log_2\left(\frac{2}{6}\right) + \frac{1}{6}\log_2\left(\frac{1}{6}\right)\right) \approx 1.459\,\text{bits} \tag{2}$$

[1] As this quantity diverges, quantum mechanical contributions dominate.

Liouville's Theorem: The conservation of classical information is well exemplified by Liouville's theorem wherein a volume in phase-space is conserved through its time evolution [2]. In a functional sense, however, classical information is lost as the coarse-graining (the union of spheres of fixed size at every point of the volume in phase-space) increases as the volume evolves, leading to the 2nd law of thermodynamics (that thermodynamic entropy in a closed system is non-decreasing with time).

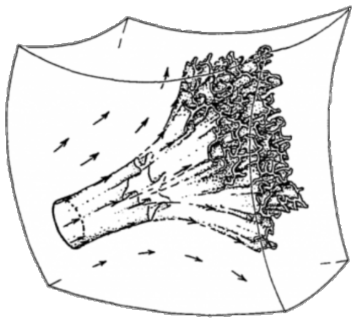

Figure 1: The evolution of a given volume in phase-space. Modified from [3].

0.1.2 Quantum Information

Quantum information is that which necessarily can only be stored in the state of a quantum mechanical system as qubits. It serves to describe physical systems on a fundamental level and quantum mechanics demands that it be conserved under all circumstances. Von Neumann entropy is the measure of quantum information.

Qubits: Qubits are the quantum version of the classical bit, defined as a two-state quantum mechanical system (e.g. that of a particle which may be spin-up or spin-down) wherein a coherent superposition allows for both states to simultaneously be realized. A qubit, like a bit, may be understood as a unit such as a meter or second, but like a bit is similarly not an SI-defined unit.

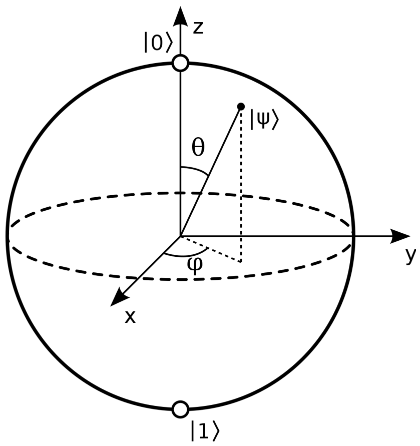

Figure 2: The Bloch sphere provides a visual representation of the qubit. A binary bit would only allow the two states represented by circles along the z-axis, while a qubit may occupy any state within the sphere [4].

Von Neumann Entropy: Von Neumann entropy is defined by:

$$S = -Tr(\rho \cdot ln(\rho)) \tag{3}$$

Where $Tr(A)$ represents the trace of a matrix "A" (the sum of elements along the diagonal of said matrix) and ρ is the density matrix (a matrix which describes the present state of a quantum mechanical system) [5].

$$\rho = \sum_j p_j \left|\psi_j\right\rangle \left\langle\psi_j\right| \tag{4}$$

S-Matrix: The conservation of quantum information for theories with asymptotic states can be shown through the unitarity (that all probabilities should sum to one) of the S-matrix (scattering matrix) [2]. The S-matrix is an array of quantities that predict the probabilities of all possible outcomes of a physical interaction [6]. Complete knowledge of an S-matrix for a physical system amounts to complete knowledge of all physical laws related to that system. For theories that lack asymptotic states (e.g. Conformal Field Theories - CFTs), no S-Matrix exists, so it is better to instead understand conservation of quantum information in terms of the time evolution of density matrices as the time evolution operator is itself unitary.

0.2 The Bekenstein Bound

The Bekenstein bound is a limit on the information content of a particular region of spacetime. As such, it is also the upper bound on the amount of information necessary to exactly describe a given physical system. At the extremal value of this boundary, a black hole is necessarily described. It should be noted that neither the classical Bekenstein bound nor the quantum Bekenstein bound were formulated until after Hawking's seminal paper on black hole radiation [7] that produced the information paradox. Bekenstein's paper on the classical bound [8] was published in 1981, 7 years after Hawking's 1974 paper [9], and Casini's paper on the quantum bound [10] was published in 2008. Despite this, a description of the Bekenstein bound serves to lay a foundation for better understanding the information paradox and the holographic principle's role in finding a solution to it.

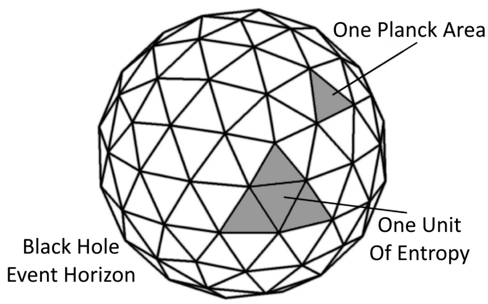

Figure 3: Visualization of the Bekenstein bound - information is encoded onto the surface of a black hole. Triangles are used for ease of tiling and have no physical significance [11].

0.2.1 Classical Bound

The classical Bekenstein bound relates Shannon entropy with mass squared, which is itself proportional to the surface area of a black hole:

$$H \leq \frac{4\pi M^2}{\ln(2)} = \frac{A}{4\ln(2)} \tag{5}$$

Where M is mass and A is surface area [12]. For example, given a Shannon entropy of $\sim 1.513 \times 10^{77}$ bits, the mass of the system in question may be no less than one solar mass $\left(\sim 1.988 \times 10^{30}\,\mathrm{kg}\right)$ and may have a surface area no less than $\sim 1.096 \times 10^{8}\,\mathrm{m}^2$ (calculated using the Schwarzschild radius of a body with one solar mass). The significance of the relation to the surface area of a black hole should be appreciated, as this is of fundamental importance to the holographic principle.

0.2.2 Quantum Bound

The quantum Bekenstein bound relates Von Neumann entropy with the modular Hamiltonian of a given region:

$$S(\rho_V) - S\left(\rho_V^0\right) \leq Tr\left(K_{\rho_V}\right) - Tr\left(K_{\rho_V^0}\right) \tag{6}$$

Where $S(\rho_V)$ is the Von Neumann entropy of the excited state, $S\left(\rho_V^0\right)$ is the Von Neumann entropy of the vacuum state, $Tr\left(K_{\rho_V}\right)$ is the trace of the modular Hamiltonian of the excited state, and $Tr\left(K_{\rho_V^0}\right)$ is the trace of the modular Hamiltonian of the vacuum state.

Modular Hamiltonians: For a given state in a quantum field theory, the local density matrix can be represented by $\rho_V = \frac{e^{-K}}{Tr(e^{-K})}$ for some Hermitian and dimensionless operator K known as the modular Hamiltonian [10].

0.3 Hawking Radiation

Hawking radiation is the process by which black holes decay over time. Discovered by Stephen Hawking in his 1974 paper [9], he described a process by which black holes may emit radiation analogously to a thermal black-body. This is inherently a quantum process and does not imply the existence of superluminal particles. There are three common ways of explaining Hawking radiation:

- The pair production origin based in quantum mechanics which exaggerates the realism of virtual particles but is generally intuitive and so used when communicating with laymen
- The Unruh origin based on the combination of the Unruh effect with the equivalence principle but which conflates a local event with a global one
- The quantum field scattering origin which is how Hawking first described the phenomenon and is ultimately the most accurate of these three pictures.

0.3.1 Pair Production Origin

A common explanation of Hawking radiation relies on the notion of spontaneous virtual pair production. In this explanation, gravitational potential energy drives the creation of entangled pairs of virtual particles in superpositions of positive energy and negative energy. These virtual particles typically collide and annihilate on a timescale of the order of the Planck-time, but when created near the event horizon of a black hole, one of the virtual particles may fall beyond the horizon. The event horizon separates it from its partner, and allows the external particle the opportunity to escape. In this process, the escaping virtual particle is redefined as a real particle (as an abuse of definitions) with positive energy. Given some imaginary boundary surrounding the black hole, the net flux experienced by this boundary will be positive, which necessarily implies that the black hole must lose mass through this process and a far-away observer would see radiation emitted from the black hole.

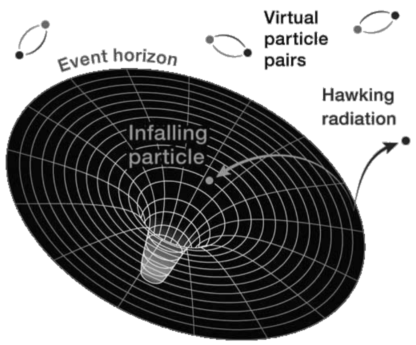

Figure 4: Visualization of Hawking radiation via the pair production origin. Modified from [13].

It is important to recognize that this description of Hawking radiation is inherently flawed in its treatment of virtual particles as if they were "real" particles. While this serves to provide a useful mental picture for readers, one should not mistake it for an exact description of the origin of Hawking radiation.

0.3.2 Unruh Origin

The Unruh effect describes a phenomenon wherein an accelerating observer will experience a thermal bath [14]. As a light cone with an origin at some distance inversely proportional to their acceleration and located opposite their direction of acceleration will be unable to coincide with them in finite time unless they cease accelerating. This gives rise to a Rindler horizon, which disrupts vibrational modes of quantum fields and theoretically gives rise to radiation observed by the non-inertial traveler. An inertial observer will not see said radiation, but will rather see a sort of "drag" applied onto the accelerating observer that increases their temperature.

The equivalence principle of General Relativity states that for an observer in free-fall, a curved spacetime reduces to a Minkowski (flat) spacetime [15]. This essentially states an equivalence between a gravitational "force" and pseudo-forces experienced in non-inertial reference frames. That is to say that an accelerating reference frame will have identical observations inherent to it as one in gravitational free-fall.

In combining these two concepts, a phenomenon similar to Hawking radiation appears. An in-falling observer will see a temperature local to the black hole. This introduces the key difference between the Unruh effect and Hawking radiation - while the Unruh effect introduces a locally observable radiation, Hawking radiation is observed far from the black hole. While the two concepts are deeply related and one can reasonably be used to better understand the other, they are ultimately distinct and should not be conflated.

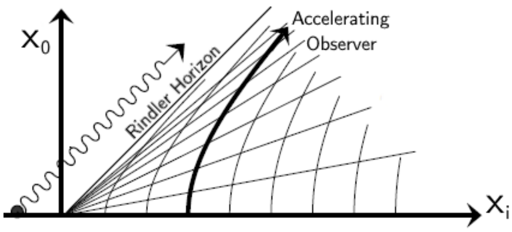

Figure 5: Visualization of the Unruh effect using a spacetime diagram. Modified from [16].

0.3.3 Quantum Field Scattering Origin

Given a quantum field in a vacuum state tracing a null geodesic (a light-like world line in Minkowski spacetime) that barely leads the formation of an event horizon (such that a photon following it is the last possible photon to avoid the black hole), the formation of the event horizon disturbs the vibrational modes of the field and gives rise to fluctuations that appear as real particles which a distant future observer receives as radiation emitted from the black hole [9]. The black hole scatters vibrational modes of the quantum field most similar to its own size, resulting in the emerging quantum field having distortions in the same wavelength range. Ultimately, this results in the radiation from the black hole having energy inversely proportional to the mass of the black hole. The energy of this radiation approximately follows a black body radiation curve, giving the black hole a temperature inversely proportional to its mass such that $T = \frac{1}{8\pi M}$.

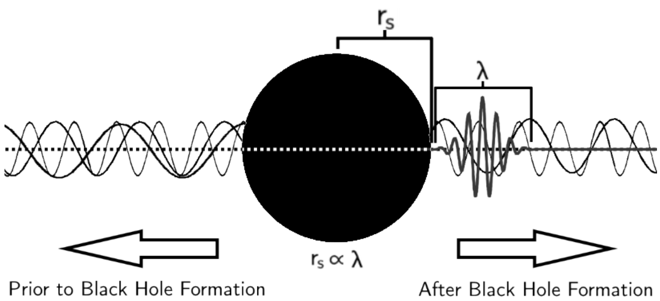

Figure 6: Visualization of Hawking radiation via the scattering of quantum fields by a horizon. Reproduced from [17].

0.3.4 The Page Curve

The Page curve describes the boundary below the intersection of the thermal entropy curve and the curve defined by $S_{BH} \sim \frac{A}{4}$ (traditionally, it has been assumed that there is exact equivalence, but this is now best avoided due to the presence of higher order corrections) for a Schwarzschild black hole [18]. This curve gives the Page time, predicting the age at which a black hole will have decayed away half of its original information content (assuming no in-falling material in the interim) [19]. The Page time is given by:

$$t_{Page} = \frac{\left[\left(\frac{\beta}{\beta+1}\right)^{3/2} - 1\right]}{\left(3M^2\dot{M}\right)} M_0^3 \approx 4786 M_0^3 \tag{7}$$

such that $M^2\dot{M} \approx -3.7474 \times 10^{-5}$ and $\beta \equiv -\dot{S}_{rad}/\dot{S}_{BH} \approx 1.48472$

At the Page time, the mass of the black hole will be:

$$M = \left[1 - \left(\frac{\beta}{\beta+1}\right)^{3/2}\right] \approx 0.77301 M_0 \tag{8}$$

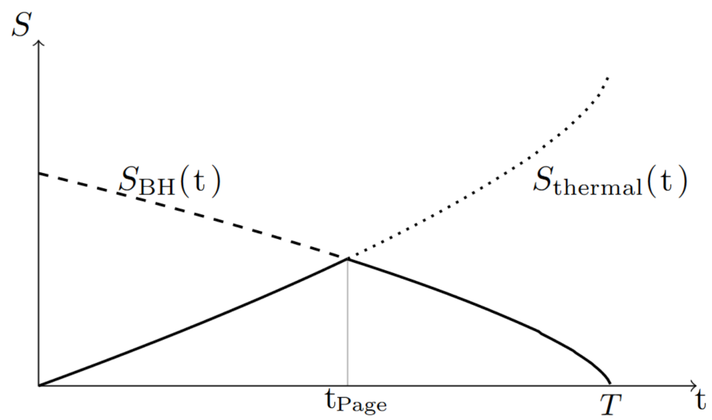

Figure 7: Illustration of the Page Curve. Modified from [18].

0.3.5 Charged and Rotating Black Holes

It should be noted that charged (Reissner-Nordström), rotating (Kerr), and charged rotating (Kerr-Newman) black holes exhibit unique particle emission spectra from Schwarzschild black holes [20]. In extremal cases (such that $a = 1$, $Q = 1$, or $a^2 + Q^2 = 1$, where a is angular momentum per unit mass and Q is charge), supergravity (a low energy limit of theories of quantum gravity as corrections to GR) becomes necessary to describe the emission spectra and to create a non-vanishing horizon.

0.4 The Information Paradox

Through the decay of a black hole via Hawking radiation, there is no established mechanism by which information contained by the black hole should be obtained by the escaping radiation. Further, by the no-hair theorem, a black hole can be fully described by an outside observer via its mass, charge, and angular momentum - no other information is observable [15]. As energy is lost to Hawking radiation, the black hole equally loses mass and must subsequently shrink. Given the Bekenstein bound, this implies a decrease in the total information content of the black hole. Essentially, this implies the loss of any information (aside from the three observable properties) that has fallen beyond the event horizon of a black hole, incurring a violation of unitarity [21].

0.4.1 No-Hair Theorem

The no-hair theorem states that a black hole can be fully described by its mass, angular momentum, and charge[2], and that no other properties are observable from the outside. This originates from the composition of the stress-energy tensor, which in turn defines the spacetime curvature and therefore the black hole. The stress-energy tensor takes the following form:

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix} \tag{9}$$

Mass contributes to this as the T^{00} component is defined as the density of mass-energy. Similarly, charge is observable because the T^{00} component for an electromagnetic field in an otherwise empty region of spacetime is, $T^{00} = \frac{\mathbf{E}^2+\mathbf{B}^2}{8\pi}$, where **E** is the electric field, and **B** is the magnetic field.[3] Angular momentum contributes to the off-diagonal components of the stress-energy tensor.

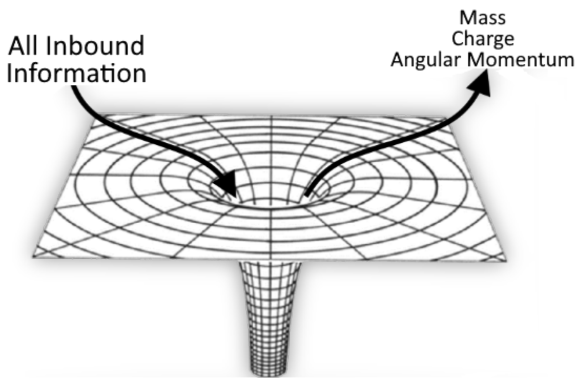

Figure 8: An illustration of the information paradox and no-hair theorem. Modified from [23].

[2]Should a magnetic monopole exist, then magnetic charge would also be an observable feature of a black hole [22]. Such a black hole would be described by a modified version of the Reissner-Nordström or Kerr-Newman metric (depending on if the angular momentum were zero or non-zero respectively) such that Q_E^2 is replaced with $Q_E^2 + Q_B^2$.

[3]More generally, $T^{jk} = \frac{1}{4\pi}\left[-\left(E^jE^k + B^jB^k\right) + \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right)\delta^{jk}\right]$, where δ^{jk} is the Kronecker delta.

Part III

Theories of Quantum Gravity

0.5 String Theory

String theory should be understood as a framework under which elementary particles are interpreted as being one-dimensional strings which extend through compactified dimensions. The type of string (e.g. open or closed), the oscillation modes of that string, their compactification, and the type of string theory in question define their particle spectra (e.g. leptons, quarks, gravitons, etc.). It began with Kaluza-Klein theory and later developed into bosonic string theory which has since expanded into five superstring theories [24]:

- Type I - contains unoriented open and closed strings in ten dimensions
- Type IIA & Type IIB - both of which contain maximal supersymmetry, closed strings, and are related by T-duality, but differ primarily in that strings in IIA are non-chiral while strings in IIB exhibit chirality
- Heterotic SO(32) & Heterotic $E_8 \times E_8$ - both of which only contain right-moving fermions (such that the $X(\tau \pm \sigma)$ solutions to the corresponding wave equations have a negative sign) but differ in that they each produce a gauge field in accordance with its name (i.e. SO(32) and $E_8 \times E_8$ respectively) in ten dimensions.

These different types of string theory are now in the process of being unified under a single UV-complete (well-behaved at arbitrary energies) M-theory. The holographic principle and AdS/CFT (Anti de-Sitter / Conformal Field Theory) correspondence, which is subsequently discussed in further detail, were derived in the context of string theories. As of yet all explicit examples of theories of quantum gravity confined to a region like an AdS spacetime which can then be described holographically on its boundary by a CFT are only consistent with string theories or have been discovered to be embeddable into string constructions. As such, while it is currently impossible to say for certain if the holographic principle and AdS/CFT correspondence are unique to string theory, they are yet to have been successfully applied to any "non-stringy" (or not at least string-adjacent as is discussed in the section on Entropic Gravity 0.7) theories of quantum gravity. It should be noted, however, that the AdS referred to in string theory is of large radius relative to the string length, and the CFT is strongly coupled. In contrast, there has been work on the development of AdS/CFT for small radii and weak coupling that may not necessarily be "stringy" in nature [25].

0.5.1 The Holographic Principle

As something falls into a black hole, an outside observer will see it freeze as it approaches the event horizon, while an in-falling observer will see themselves cross the horizon [2]. This results in a copying of information wherein one copy is present on the event horizon of the black hole, while the other is internal to it. This is not, however, a violation of the quantum no-cloning theorem (wherein it is impossible to create an independent copy of an arbitrary quantum state), as the copies are disconnected - no observer can ever see both. This is known as black hole complementarity. By allowing information to remain coalesced at the event horizon, we come to find the 3D structure of the black hole may be entirely described by this 2D surface boundary. This can be extended to find a lower dimensional description of gravity throughout the entire universe.

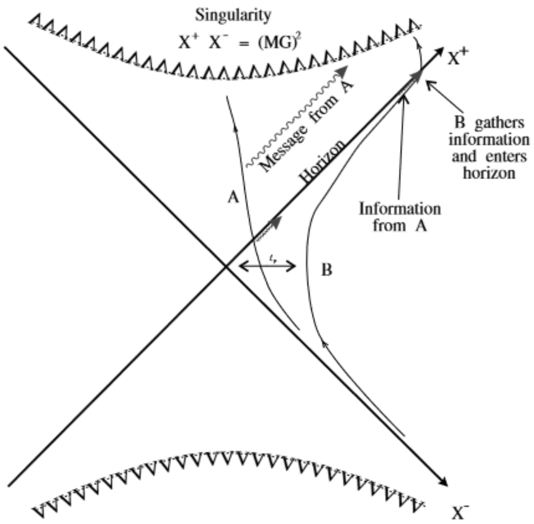

Figure 9: Kruskal-Szekeres coordinates diagram depicting how black hole complementarity does not result in a violation of the no-cloning theorem. [2].

In the above illustration: upon crossing the horizon, the information from "A" remains at the boundary to later be received by "B". "B" hovers some distance from the black hole and collects this information from Hawking radiation, then crosses the horizon. Despite both "A" and "B" entering the black hole, no transmission by "A" can ever reach "B" to provide them with a duplicate of the information the latter has already received. No information from "A" is emitted by the black hole until ($t^* \approx M^3$), and we can assume that after hovering above the horizon,"B" enters the black hole at light cone coordinates $x^{\pm} = \rho e^{\pm\omega}$ (wherein $x^{\pm}$ are lightcone coordinates, ρ is the proper spatial distance of the particle from the horizon, and ω is a time-like coordinate referred to as the Rindler time) with:

$$x_*^+ x_*^- > \ell_P^2 \tag{10}$$

and

$$x_*^+ \gtrsim \ell_P e^{M^2} \tag{11}$$

The subscript "$*$" denotes a particular value of the lightcone coordinates as given by $\omega^* \geq \frac{t^*}{4M}$ and $\rho^* \geq \ell_P$.
Given that the singularity is at $x^+x^- = M^2$, observer "B" will collide with it at $x^- \lesssim Me^{-M^2G}$, giving a signal from "A" a limited amount of time to reach "B":

$$\Delta t \approx Me^{-M^2} \tag{12}$$

It then follows that such a signal must have energy:

$$E \gtrsim M^{-1}e^{M^2} \tag{13}$$

Thereby requiring that "A" have a mass-energy significantly larger than the black hole, which necessarily means that "A" can not fit within the horizon. As such, no signal can exist such that "B" can obtain the information that was left at the event horizon and then receive a duplicate copy inside the black hole, thereby preserving the no-cloning theorem within black hole complementarity [2].

0.5.2 AdS/CFT Correspondence

The most successful realization of the holographic principle is the AdS/CFT correspondence. In this conjecture, we have the "AdS side" with weakly interacting perturbative gravity in D-dimensions with a constant negative curvature, and the "CFT side" with a strongly interacting non-perturbative Weyl-invariant (as the Polyakov action is unchanged by a local rescaling of the metric tensor) quantum field theory of (D-1)-dimensions, which is necessarily unitary [26]. While this defines characteristics of such a CFT, there remains a multitude of possible CFTs that satisfy this correspondence - for example, the Chern-Simmons action in AdS requires a counter-term that can be interpreted as the action for such a CFT. The existence of a unitary CFT that is dual to a horizon in AdS assures that Hawking radiation preserves unitarity within an AdS spacetime. The most notable short-coming of this theory is that our universe has de-Sitter (dS) spacetime, such that it has a constant positive curvature (it is spatially flat, but when time is accounted for, the geometry is ultimately positively curved).

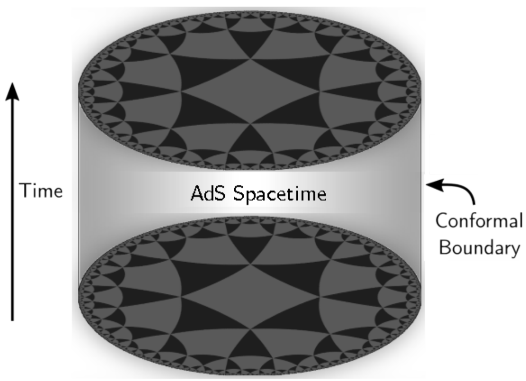

Figure 10: Illustration of AdS_3 spacetime with a conformal boundary at infinite spatial distance. Modified from [27].

Beyond AdS/CFT

There are active efforts to expand or refine AdS/CFT correspondence to be more physically realistic. Two of the more notable efforts to do so are dS/CFT correspondence and Kerr/CFT correspondence [28].

dS/CFT Correspondence: dS/CFT correspondence is an ongoing effort by Strominger et al. to develop an analog to the AdS/CFT correspondence for a de-Sitter spacetime such that it would have descriptive power within out own universe [29]. Within this theory, the conformal metric for the CFT that exists at a spatial boundary at infinity in AdS is manipulated such that it can apply instead to a dS. The AdS length is multiplied by *i*, a spatial coordinate has a Wick rotation applied to it to make it time-like, and a Minkowski space-like coordinate is made time-like [30]. In doing this, the CFT is moved from spatial infinity to time-like infinity.

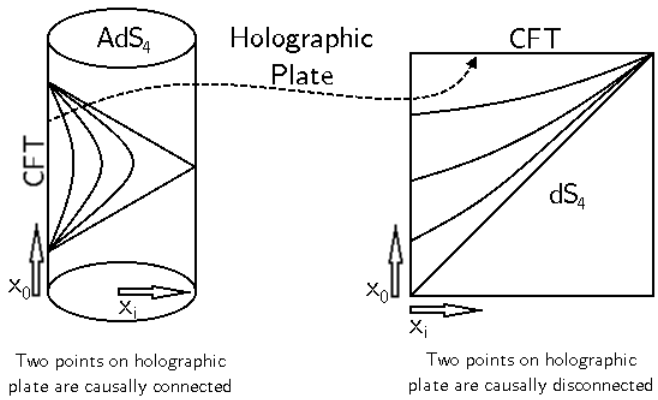

Figure 11: Illustration of dS_2 spacetime with a conformal boundary at infinite temporal distance. Reproduced from [30].

Kerr/CFT Correspondence: As Schwarzschild black holes are not physically realistic (except perhaps in the case of black holes that have almost entirely decayed), it is worthwhile to consider the more realistic case described by the Kerr metric. This leads to an extension of AdS/CFT known as Kerr/CFT [28]. Originally formulated with respect to extremal Kerr black holes, the theory was later extended to contain black holes with lower angular momentum.

0.5.3 Open Problems in String Theory

There are presently multiple open problems within string theory. Perhaps most notable is that string theory lacks a background independent description and there are few backgrounds in which strings can currently be quantized. In other backgrounds, perturbations can be applied around flat space, but these corrections fail as gravity approaches its upper strength limit. The unification of the different types of string theories into one M-theory is also currently incomplete.

0.6 Loop Quantum Gravity

Loop Quantum Gravity (LQG) posits that the universe should be understood as being comprised of a finite number of Planck-scale loops into what is then observed as a granular space with gravity as an embedded feature of that space, resulting in a background independent theory of quantum gravity. This collection of loops is called a "spin network"[31]. Unlike string theory, LQG can preserve General Relativity's geometric formulation of gravity rather than considering it as a force.

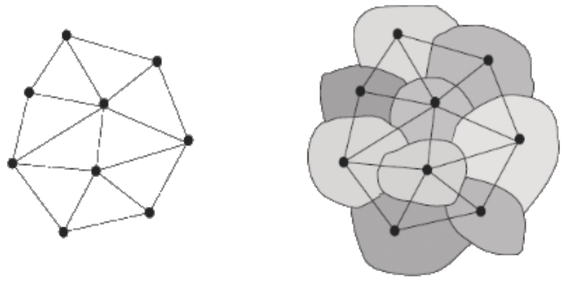

Figure 12: A spin network and the discretized space regions created by it. Modified from [32].

There exist two competing variants of the theory - canonical LQG and covariant LQG. The former approach involves defining elements of a Hamiltonian constraint operator acting on a spin network, and then defining the kernel (the set of all vectors sent to zero by this operator) to be the associated Hilbert space. The structure of said Hilbert space is not well-understood at this time. The latter approach introduces the notion of the "spin foam", which is the four dimensional extension of the spin network as it evolves over time - for this reason, covariant LQG is also known as the "spin foam model" [33]. Presently, only the covariant form has been successful in reproducing the Einstein Field Equations in its semi-classical limit, but does suffer from infrared divergences.

0.6.1 Planck Stars

LQG offers a potential solution to the information paradox via a hypothetical object known as a "Planck star" - Proposed by Carlo Rovelli and Francesca Vidotto, it is a highly dense object consisting of some sort of exotic (read: non-baryonic) matter interior to a black hole. A Planck star is created upon a collapsing object reaching the Planck energy density interior to a black hole. Under the assumption of a quantized spacetime as found in LQG, such an energy density would result in a repulsive force counter to the gravitational collapse, as further compression would necessitate a violation of the uncertainty principle. This essentially erases the information paradox as it creates an internal structure to black holes - the entire volume into which information may be encoded [34]. While such information is temporarily inaccessible, it is in principle preserved and will ultimately rejoin the external universe in sufficient time.

0.6.2 Open Problems in Loop Quantum Gravity

LQG presently faces multiple problems: As LQG creates a lattice, chiral fermionic fields placed on such a space may exhibit fermion doubling due to the Nielsen-Ninomiya theorem if the lattice exhibits translational symmetry, wherein more fermionic states than expected emerge - various solutions have been proposed (e.g. modified fermions), but it is still considered an open problem as it is unclear if this phenomenon affects LQG or if a resolution may be found. Within canonical LQG, there are ambiguities within the Hamiltonian constraint. It has also not been concretely proven that spin foams are necessarily the correct description of a spin network's time evolution - further, there are many models of spin foam gravity, although the EPLR (Engle, Pereira, Rovelli, & Livine) model has reached general consensus as the most viable at this point in time due to its ability to derive the EFEs in its semiclassical limit [35].

0.7 Entropic Gravity

Entropic Gravity (i.e. emergent gravity) attempts to describe gravity as an emergent property of local entropy (not unlike gas pressure). Originating from Bekenstein and Hawking's findings on black hole thermodynamics which imply an inherent connection between gravity and entropy, Ted Jacobson showed in 1995 that the Einstein field equations can in principle be derived from thermodynamic laws [36]. Erik Verlinde then found in 2011 that the change in entropy per *n* bits of information can be expressed in terms of the change in the Newton potential such that:

$$\frac{\Delta S}{n} = -k_B \frac{\Delta \Phi}{2c^2} \tag{14}$$

where Φ can be understood in terms of the Killing vector ξ^a: $\Phi = \frac{1}{2} log\left(-\xi^a \xi_a\right)$. This naturally leads to the Einstein Field Equations and lends to the application of the holographic principle in which the Newton potential plays the role of a course graining variable analogous to the scale in AdS/CFT in which the gradient of the Newton potential gives the holographic direction. In essence, this is a model in which gravity is a consequence of information densities [37]. It should be noted that the existence of closed strings (which predicate gravitons) from a string theory are viewed as an effective description within Entropic Gravity as an emergent phenomenon, rather than as fundamental objects as in string theory. Regardless of the physical realness of closed strings, the holographic principle can be applied to Entropic Gravity, then resolving the information paradox via AdS/CFT.

0.7.1 Open Problems in Entropic Gravity

Entropic Gravity has faced multiple criticisms - perhaps most notably by Zhi-Wei Wang and Samuel Braunstein, where in their 2018 paper they found that while spacetime in the vicinity of spacetime horizons are "truly thermodynamic", general spacetime surfaces that are not spherically symmetric and do not exhibit horizon-like behavior fail to obey an analog to the first law of thermodynamics - thereby undermining an assumption of Entropic Gravity [38]. Despite this flaw, research into Entropic Gravity continues as an effective model within some yet undefined limit.

0.8 Causal Dynamical Triangulation

Causal Dynamical Triangulation (CDT) seeks to act as a nonperturbative path-integral description of quantum gravity that is background independent. In this description, spacetime is divided into 4-simplex structures (a 5-vertex four-dimensional object bounded by five tetrahedral cells) (also referred to as 5-cells). While individually flat, these 4-simplexes connect together to form a spin foam like found in LQG which is macroscopically viewed as a Lorentzian manifold. The 4-simplexes may only be connected such that the arrow of time of conjoined edges are in agreement. It is the respective orientations of these 4-simplexes that then determine the observed curvature of the local spacetime. The CDT path integral formulation and its corresponding action are as follows:

$$Z^{CDT}(G_N,\Lambda) = \lim_{a\to 0} \sum_{T\in\mathcal{T}} \frac{1}{C(T)} e^{iS^{CDT}[T]} \tag{15}$$

$$S^{CDT}[T] = G_b\pi\sqrt{4\alpha+1}N_0(T) + \mathcal{A}(\alpha, G_b, \Lambda_b)N_4^{(4,1)}(T) + \mathcal{B}(\alpha, G_b, \Lambda_b)N_4^{(3,2)}(T) \tag{16}$$

Wherein the limit as $a \to 0$ should be understood with respect to the geodesic length of the time-like edges (ℓ_s) and the space-like edges (ℓ_t) wherein $\ell_s \equiv a$ and $\ell_t^2 = -\alpha a^2$, the sum is taken over inequivalent Lorentzian triangulations (T) assembled from two types of 4-simplex in accordance with the prior-mentioned causal gluing rule. $C(T)$ is the count of elements in the automorphism group of T. $N_i^I(T)$ denotes the number of simplexes of dimension $0 \leq i \leq 4$ while I denotes which of the two subtypes of 4-simplex is being described. $\mathcal{A}$ and $\mathcal{B}$ are respectively linear combinations of the bare Newton constant (which can be understood as a corrective term on the vacuum value for the Newton constant to make it fit our observed value) and the bare cosmological constant (as before, but for the cosmological constant). $0 < \alpha < \infty$ is a fixed parameter describing the geodesic length of a time-like edge.

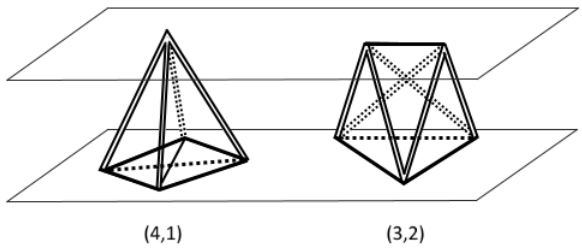

Figure 13: The two forms of 4-simplexes in CDT: Space-like edges are single lines while time-like edges are double lines. Modified from [39].

It has been shown that the expectation value of the global shape of such a dynamically generated universe resembles that of a de Sitter universe. As a consequence of this theory, spacetime contracts to a two-dimensional hologram at the Planck scale. Unlike other theories of quantum gravity, CDT contains a well-defined Wick rotation, which allows for powerful numerical methods to be applied, allowing for a great deal of depth in probing the predictions of this theory [39]. Monte-Carlo simulations of CDT have proven successful in reproducing various predictions of semi-classical gravity (e.g. Hawking radiation). Despite these strengths, CDT does not currently offer a solution to the information paradox. The current priority in CDT research is using these numerical methods to establish a continuum limit of the theory with hopes that this limit reproduces GR. There have been efforts to combine CDT with Verlinde's work on Entropic Gravity such that particles and fields should be emergent from the structure and evolution of the spin foam. Similarly, CDT exhibits various similarities with LQG and in its 2D continuum, its Hamiltonian appears to be the same as for Hořava-Lifshitz Gravity in 2D [40].

0.8.1 Open Problems in Causal Dynamical Triangulation

Like with LQG, its is unclear if CDT suffers from the fermion doubling problem 0.6.2 as CDT may lack the necessary translational invariance for the Nielsen-Ninomiya theorem due to random triangularization.

0.9 Hořava-Lifshitz Gravity

Hořava-Lifshitz Gravity (HLG) is a potentially UV-complete approach to quantum gravity in which quantum field theory's treatment of time is considered more fundamental than General Relativity's treatment of time as a dynamic dimension. At the Planck scale, the speed of light goes to infinity, but to maintain consistency with predictions of General Relativity, a finite speed of light and Lorentz invariant time emerge at large distances. The lapse function (*N*) is a Lagrange multiplier belonging to the Arnowitt-Deser-Misner (ADM) variables, which collectively serve to describe how the foliations of spacetime are connected within the ADM Hamiltonian formulation of GR. Assuming parity and a time-dependent lapse function, the "minimal theory" action for Hořava-Lifshitz gravity can be expressed as:

$$S_g = \zeta^2 \int dt d^3x N \sqrt{g} (\mathcal{L}_K - \mathcal{L}_V) \tag{17}$$

Wherein ζ^2 is the gravitational coupling constant, g is the determinant of the 3d metric tensor of the foliations, $\mathcal{L}_K$ is the kinetic component of the Lagrangian, and $\mathcal{L}_V$ is the potential component of the Lagrangian.

The de Sitter spacetime in this "minimal theory" is stable and in the IR-limit this action can be reduced to the Einstein-Hilbert action. This "minimal theory" requires non-perturbative methods for phenomenological analysis, but in doing so with spherical symmetry in static vacuum configurations, it was found to be consistent with GR. HLG does not currently provide a solution to the information paradox.

0.9.1 Open Problems in Hořava-Lifshitz Gravity

HLG currently lacks a proof of its renormalizability outside of a select few specific cases. The specific details of how and if this theory can approach GR in the IR-limit are presently lacking. It is also unclear how this theory behaves when matter is introduced into it [41].

0.10 Twistor Theory

Twistor Theory is a formalism proposed by Roger Penrose in which there exists a complex vector space (twistor space) of solutions of the zero rest mass equation $\nabla^{(A}_{A'}\Omega^{AB\dots)} = 0$. Here, ∇ denotes a two-component complex column vector - the spin connection which is derivable from the Levi-Civita connection in Minkowski spacetime, but otherwise only derivable from the more general affine connection which is not necessarily torsion-free. $\Omega^{AB\dots}$ is the spinor field describing a particle of spin S wherein the Latin indices can take values of 0 or 1. In this form, the equation describes a zero rest mass particle with S equal to half of the number of indices (e.g. $\nabla^{(A}_{A'}\Omega^{ABCD)} = 0$ describes the graviton) [42]. In essence, this replaces the dynamically curving spacetime with a twistor space that offers additional freedom in hopes that other established theories such as GR could be recast in this formalism in an attempt to make a quantum description more accessible. Twistor Theory, while important as a means of constructing amplitudes in ABJM QFT to provide a holographic dual to M-theory in $AdS_4 \times S^7$ has otherwise fallen out of favor as a pathway to the development of a theory of quantum gravity, yet remains significant for the mathematical insights it provides. It does not provide any insight into a solution for the information paradox.

Part IV

The Complementarity Paradox

It has been found that Hawking radiation emitted about the Page time should be entangled with all prior emissions from the black hole and the black hole itself [43]. This results in a violation of the principle of monogamy of entanglement, wherein a particle can not be entangled with two other independent particles simultaneously. There are multiple proposed solutions to this paradox as outlined below.

0.11 Null Hypothesis

The first notable resolution to this paradox is the null hypothesis, or alternatively, that there simply is no paradox in need of a resolution [44]. This demands an abandonment of unitarity, which results in black hole information loss. As such, this is an unpopular hypothesis.

0.12 Firewalls

Black hole firewalls (also known as AMPS firewalls) are an emergent property of black holes from the proposal that the entanglement between emitted particles should be immediately broken [43]. This results in the expenditure of a significant amount of energy, creating a hot barrier at the event horizon. This demands an abandonment or modification of Einstein's equivalence principle, as such a barrier could not exist in empty space.

0.13 Fuzzballs

Fuzzballs are a proposed alternative to black holes, wherein the event horizon is replaced with a highly condensed surface of strings and the spacetime interior to a black hole is replaced with a vacant region - thereby increasing the topological genus of spacetime [45]. This structure eliminates the no-hair theorem and in doing so resolves the information paradox without need for black hole complementarity. However, fuzzballs do not as of yet have an associated theory of quantum gravity that predicts their existence.

0.14 ER=EPR

The ER=EPR proposal suggests that non-traversable wormholes (Einstein-Rosen bridges - ER) connect entangled particles (Einstein-Podolsky-Rosen - EPR), and thereby are not independent systems [46]. This allows for the conservation of information while retaining the equivalence principle.

Part V

Conclusion

In the analysis of these different approaches to the development of a theory of quantum gravity, I have found the degree of how interconnected many of these theories are to be noteworthy. As previously mentioned, Entropic Gravity is rooted in string theory, Twistor Theory is significant to M-theory in $AdS_4 \times S^7$, HLG appears to have the same Hamiltonian as CDT in 2d, and the similarities in lattice structure between CDT and LQG cannot be overstated. This observation leaves me with the potentially naïve conclusion that there is perhaps a deeper theory which may be ultimately discovered and takes advantage of various properties of each of these current contenders. Furthermore, it is conceivable that our picture of classical gravity is simply incomplete and that in the pursuit of explanations for phenomena like dark matter and dark energy, we may yet discover a deeper classical description of gravity that is necessary for the development of this ultimate theory of quantum gravity.

Of the various solutions to the information paradox that I explored through this thesis, I have found the AdS/CFT correspondence to be the most compelling. Even if string theory itself should eventually find an impasse in its development, I would not find it at all surprising if the AdS/CFT correspondence (or perhaps someday a dS/CFT correspondence) were to be a feature of the ultimately correct theory of quantum gravity.

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