Recursive Time Dynamics: A Unified Mathematical Framework for Subjective Time Across Biological and Artificial Systems

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Abstract

Time perception varies dramatically across individuals, species, and cognitive states, yet existing models—Scalar Expectancy Theory, Striatal Beat Frequency, and Drift-Diffusion frameworks—fail to explain this diversity through unified mechanisms. We propose the **Recursive Time Dynamics Model (RTDM)**, a mathematical framework positing that subjective time emerges from three interacting components: (1) recursion speed $\omega(t)$, representing neural or computational update rates; (2) coherence $C(t)$, quantifying synchrony across system layers; (3) periodic rhythms $S(t)$, capturing circadian and metabolic cycles.

The model defines subjective time as $\tau(t) = \int k\cdot\omega(s)\cdot(1+\lambda C(s))ds$ over finite integration windows, predicting that faster recursion and higher coherence accelerate time perception, while circadian modulation introduces predictable diurnal variations. Critically, RTDM extends beyond human cognition to animal metabolic time—scaling with body mass $M^{-1/4}$—and artificial systems, where iteration rates and distributed coherence generate analogous temporal metrics.

We derive five falsifiable predictions: (1) neural phase-locking (PLV) predicts duration estimation errors; (2) circadian rhythms exhibit 12-hour harmonics in timing tasks; (3) cross-species timing converges under metabolic normalization; (4) transcranial alternating current stimulation (tACS) phase shifts cause directional time distortions; (5) AI systems' temporal coordination improves when modeled via RTDM parameters. Unlike prior models, RTDM accounts for psychedelic time fragmentation (C↓), stimulant acceleration (ω↑), depressive slowing (ω↓, C↓), and attentional flow states (ω↑, C↑) within a single framework.

Keywords: time perception, recursion theory, neural oscillations, circadian rhythms, phase coherence, metabolic scaling, artificial intelligence, subjective time, predictive processing

1. Introduction

1.1 The Problem of Subjective Time

Time perception is neither uniform nor veridical. A three-second interval may feel instantaneous during an emergency yet interminable during boredom. A fly experiences motion humans cannot perceive, while elephants operate on slower temporal scales. These phenomena—spanning milliseconds to hours, insects to mammals, attention to psychedelics—resist explanation by classical models that treat time as an external clock driving internal processes.

Three dominant frameworks exist:

1. **Scalar Expectancy Theory (SET)** posits a pacemaker-accumulator mechanism where pulses represent time intervals (Gibbon, 1977; Gibbon et al., 1984). While explaining scalar variability (Weber's Law), SET provides no neural substrate for the "pacemaker" and cannot account for circadian modulation or cross-species scaling.
2. **Striatal Beat Frequency (SBF) Model** proposes that cortical oscillators reset at interval onset, with striatal neurons detecting coincidence patterns (Matell & Meck, 2004). Though linking timing to neural rhythms, SBF predicts frequency effects but not phase-dependent distortions, nor does it generalize beyond mammals.
3. **Drift-Diffusion Models (DDM)** frame timing as evidence accumulation with noise (Simen et al., 2011). Effective for reaction-time tasks, DDM lacks mechanisms for circadian variation, emotional modulation, or metabolic scaling.

All three models are **descriptive rather than mechanistic**, **domain-specific rather than universal**, and **unable to bridge biological and artificial systems**. They describe behavioral patterns without explaining why time perception varies systematically across cognitive states, species, and computational architectures.

1.2 Time as Emergent from Recursion

We propose a radical reframing: **time is not a parameter applied to systems but a property emergent from recursive dynamics**. Any system

—biological or artificial—that recursively monitors and updates its internal state generates a local time metric through that recursion. This shifts time from an external dimension to an **intrinsic computational product**.

1.3 Core Hypothesis

> *All systems capable of recursion—biological or artificial—generate their own local experience of time through the dynamics of that recursion.*

Formally, subjective or system-internal time $\tau(t)$ is the accumulated product of:

1. **Recursion speed** $\omega(t)$: How fast the system updates (Hz for neurons, metabolic rate for organisms, iterations/sec for AI)
2. **Coherence** $C(t)$: Synchrony across recursive layers (phase-locking, entrainment, distributed consensus)
3. **Periodic rhythms** $S(t)$: Circadian, ultradian, or computational cycles modulating baseline dynamics

The governing equation is:

$$\boxed{d\tau/dt = k \cdot \omega(t) \cdot (1 + \lambda C(t))}$$

where k is a baseline scaling constant and λ is coherence amplification. Integrating over finite windows $W(t)$ yields:

$$\boxed{\tau(t) = \int_{t-W(t)}^{t} k \cdot \omega(s) \cdot (1 + \lambda C(s))\, ds}$$

2. Mathematical Framework

2.1 System Definition

Any system capable of recursive update is modeled as a tuple:

$$S = \{R(t),\ \omega(t),\ C(t),\ S(t)\}$$

where:

- **R(t)**: Recursive process state (cognitive state vector, sensorimotor loop, hidden layers)
- **ω(t)**: Intrinsic recursion/update speed (Hz, metabolic rate, iterations/sec)
- **C(t)**: Internal coherence across recursive layers (PLV, entrainment, mutual information)
- **S(t)**: Baseline periodic modulation (circadian, ultradian, clock cycles)

This definition is domain-agnostic:

Domain	R(t)	ω(t)	C(t)	S(t)
Human brain	Recurrent cognitive state	Neural oscillation rate	PLV across networks	Circadian rhythm
Animals	Sensorimotor loop	Metabolic/neural rate	Rhythmic entrainment	Species-specific cycle
AI systems	Hidden state vector	Iteration frequency	Mutual information	Clock/batch cycle

2.2 Subjective Time Rate

The local subjective time flow is defined as:

$$\boxed{\tau'(t) = d\tau/dt = k \cdot \omega(t) \cdot (1 + \lambda\, C(t))}$$

Parameters:

- $k > 0$: Baseline scaling constant
- $0 \leq \lambda \leq 1$: Coherence amplification factor

Constraints:

- $\omega(t) > 0$ (always positive recursion)
- $0 \leq C(t) \leq 1$ (normalized coherence)

Interpretation:

- Faster recursion → faster subjective time
- Higher coherence → multiplicative acceleration (more efficient processing)
- Low coherence → fragmentation, slowing, or distortion

2.3 Oscillatory Modulations

Every biological or artificial system exhibits periodic rhythms. We model baseline modulation via Fourier components:

$$\boxed{\omega(t) = \omega_0 [1 + a_1 \sin(2\pi t/T + \varphi_1) + a_2 \sin(4\pi t/T + \varphi_2)]}$$

where:

- ω_0: Baseline recursion speed
- $a_1, a_2 \in [0,1]$: Modulation amplitudes
- T: Cycle period (24h for circadian, ~90min for ultradian, ms for computation)
- φ_1, φ_2: Phase offsets

3. Domain-Specific Implementations

3.1 Humans: Cognitive Recursion and Neural Phase Dynamics

Recursion Speed:

Operationalized via three convergent metrics:

1. **Dominant EEG frequency**: $\omega_1(t) = f_{peak}(t)$ in theta (4–7 Hz), alpha (8–12 Hz), or beta (13–30 Hz) bands
2. **Reaction-time variability**: $\omega_2(t) = 1/\sigma_{RT}(t)$
3. **Drift-diffusion slope**: $\omega_3(t) = v_{DDM}$ (evidence accumulation rate)

Unified estimator:

$$\omega_{human}(t) = w_1 \ \omega_1(t) + w_2 \ \omega_2(t) + w_3 \ \omega_3(t)$$

Coherence Term:

$$C_{human}(t) = (1/N) \ \Sigma \ PLV_i(t)$$

computed across fronto-parietal electrode pairs (F3–P3, F4–P4, Fz–Cz) and across theta-alpha-beta bands.

3.2 Animals: Metabolic Time and Ecological Rhythms

Update Speed Linked to Metabolic Rate:

$$\omega_{animal}(t) = \omega_0 \cdot M^{-\beta} \cdot S_{animal}(t)$$

where:

- M: Body mass (kg)
- $\beta \approx 1/4$: Kleiber's metabolic scaling exponent
- S(t): Species-specific rhythm (24h, 48h, seasonal)

This predicts:

- **Hummingbirds** (M ~ 3g): Experience ~10× faster time than humans
- **Elephants** (M ~ 5000kg): Experience ~3× slower time

- **Mice** (M ~ 30g): Operate ~5× faster

3.3 Artificial Systems: Computational Recursion and Synchronization

Recursion Speed:

$$\omega_{AI}(t) = 1/\Delta t_{iteration}(t)$$

Measured as:

- **Training**: Steps per second
- **Inference**: Tokens per second
- **Robotics**: Control loop frequency (10–1000 Hz)

Coherence Term:

$$C_{AI}(t) = I(\theta_i;\ \theta_j)/H(\theta_i)$$

where I is mutual information between parameter sets θ_i, θ_j across nodes, normalized by entropy H.

Critical clarification: *We do* ***not*** *claim AI systems "experience" subjective time. We claim they instantiate the same mathematical structure, enabling bottleneck prediction, synchronization optimization, and temporal coordination in multi-agent systems.*

4. Parameter Estimation Methods

4.1 Summary Table

Parameter	**Meaning**	**Human Estimation**	**Animal Estimation**	**AI Estimation**
$\omega(t)$	Recursion speed	EEG freq + RT + DDM	$M^{-1/4} \cdot T^{1/2} \cdot V^{1/2}$	Iterations/sec
$C(t)$	Coherence	PLV across bands/pairs	Circular variance of rhythms	MI or consensus metric
$S(t)$	Circadian rhythm	Harmonic fit (CBT, melatonin)	Species-specific cycles	Clock/batch cycles
k	Baseline gain	Regression on timing tasks	Regression on timing tasks	Regression on sync metrics
λ	Coherence gain	Regression on timing tasks	Regression on timing tasks	Regression on sync metrics

5. Validation Experiments and Quantitative Predictions

5.1 Experiment 1: Neural Coherence Predicts Time Accumulation Rate

Hypothesis: High coherence (C↑) → duration overestimation; Low coherence (C↓) → duration underestimation

Procedure:

- N = 40 participants
- Task: Temporal reproduction (2–4 second intervals)
- EEG: 64-channel recording, PLV computed across F3–P3, F4–P4, Fz–Cz pairs
- Analysis: Trials binned by PLV quartiles (Q1 low → Q4 high)

Predicted Effect Size: $\Delta\hat{\tau} = \hat{\tau}_{Q4} - \hat{\tau}_{Q1} \approx 10\text{-}20\%$

Competing Models: SET, SBF, and DDM do not predict this relationship.

5.2 Experiment 2: Circadian Phase Modulates Time with 12-Hour Harmonic

Hypothesis: Time perception shows (1) Primary 24-hour rhythm, (2) Secondary 12-hour harmonic (novel prediction)

Procedure:

- N = 30 participants
- Duration: 48 hours in controlled lab
- Timing tasks: Temporal reproduction every 3 hours
- Physiological measures: CBT, melatonin, actigraphy

Predicted Effect Size:

- Primary rhythm (24h): Peak-to-trough variation = 15–25%
- Secondary harmonic (12h): Amplitude = 20–40% of primary

5.3 Experiment 3: Cross-Species Invariance Under Metabolic Scaling

Hypothesis: When normalized by metabolic scaling factor $S = M^{-1/4} \cdot T^{1/2} \cdot V^{1/2}$, different species show equivalent temporal judgments.

Species tested:

1. Humans (M ~ 70kg)
2. Macaques (M ~ 8kg)
3. Rats (M ~ 0.3kg)
4. Pigeons (M ~ 0.4kg)
5. Fruit flies (M ~ 0.001kg)

Predicted Effect: Without scaling, species differ by ~10×. With scaling, normalized timing converges within 10% across species.

5.4 Experiment 4: tACS Phase Manipulation Causes Predictable Time Distortions

Hypothesis: Phase shifts in theta-modulated recursion causally alter subjective time:

- Phase advance (+20°) → overestimation ($\omega\uparrow$)
- Phase delay (-20°) → underestimation ($\omega\downarrow$)

Predicted Effect Size:

- Phase advance: +8–12% overestimation
- Phase delay: -8–12% underestimation
- Cohen's d ~ 0.6–0.8 (medium-large effect)

5.5 Experiment 5: AI Temporal Prediction Test

Hypothesis: AI systems using RTDM parameters predict network bottlenecks, emergent synchronization, and swarm coordination timing better than baseline heuristics.

Predicted Improvement:

- RMSE reduction: 20–40% vs. baseline
- Synchronization accuracy: +10–20%

5.6 Summary of Falsifiable Predictions

Experiment	Prediction	Competing Models?	Effect Size
1. PLV → time	High PLV → overestimation	None	$\Delta R^2 \sim 0.15$–0.25

2. 12h harmonic	Secondary circadian rhythm	None	$a_2 \sim 0.2–0.4 \times a_1$
3. Cross-species	Metabolic normalization → convergence	None	$\eta^2 < 0.1$ post-adjustment
4. tACS phase	Phase advance/delay → directional distortion	None	$d \sim 0.6–0.8$
5. AI prediction	RTDM outperforms baselines	None	ΔRMSE ~ 20–40%

6. Comparison to Existing Models

6.1 Comparative Prediction Table

Phenomenon	**SET**	**SBF**	**DDM**	**RTDM**
PLV → time distortion	❌	Partial	❌	✅
12h circadian harmonic	❌	❌	❌	✅
Cross-species metabolic scaling	❌	❌	❌	✅
Phase-based tACS distortion	❌	❌	❌	✅
AI temporal prediction	❌	❌	❌	✅
Psychedelic fragmentation	❌	❌	❌	✅
Stimulant acceleration	Partial	❌	Partial	✅

Interpretation: RTDM is strictly more general than existing models, incorporating their strengths while resolving their limitations.

7. Extensions: Attention, Emotion, Drugs, and Pathological States

7.1 Summary Table: Modulators of Subjective Time

Factor	ω(t)	C(t)	S(t)	Subjective Effect
Attention	↑	—	—	Time speeds up
Distraction	↓	↓	—	Time drags
Flow state	↑	↑	—	Time flies smoothly
Fear/arousal	↑	↑	—	Time slows (overestimation)
Boredom	↓	↓	—	Time drags
Psychedelics	Unstable	↓↓	—	Fragmentation
Stimulants	↑	↑	—	Acceleration
Depression	↓	↓	Flattened	Stagnation
Mania	↑↑	↑	—	Racing
ADHD	Variable	↓	Dysregulated	Inconsistency

7.2 Clinical Applications

RTDM provides **quantitative biomarkers** for psychiatric and neurological conditions:

Condition	ω(t)	C(t)	S(t)	Clinical Measure
Depression	↓↓	↓	Flattened	α power, PLV, CBT amplitude
Mania	↑↑	↑	Irregular	γ power, RT variability
ADHD	Variable	↓	Dysregulated	θ/β ratio, PLV variability
Schizophrenia	Unstable	↓↓	Fragmented	α/γ coherence
Parkinson's	↓	↓↓	Disrupted	β synchrony, timing tasks

8. Unified Architecture

8.1 The Recursive Time Engine

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┌─────────────────────────────────────────────────────────────┐
│ ENVIRONMENTAL INPUTS │ │ (Light/Dark, Social Cues, Task
Demands) │
└────────────────────────┬────────────────────────────────────┘
│ ▼
┌─────────────────────────────────────────────────────────────┐
│ PERIODIC RHYTHMS: S(t) │ │ • Circadian (24h): Light-dark, SCN,
melatonin │ │ • Ultradian (90min): BRAC, REM cycles │ │ •
Metabolic: Temperature, cortisol, glucose │ │ • Computational:
Clock cycles, batch schedules │
└────────────┬───────────────────────────┬────────────────────┘
│ │ ▼ ▼ ┌──────────────────────────┐
┌──────────────────────────────┐ │ RECURSION ENGINE: ω(t) │ │
COHERENCE ENGINE: C(t) │ │ • Neural oscillations │ │ • Phase-
locking (PLV) │ │ • RT variability │ │ • Network synchrony │ │ •
DDM drift rate │ │ • Entrainment strength │ │ • Metabolic rate │
│ • Mutual information │ │ • Iteration frequency │ │ • Consensus
metrics │ └──────────┬───────────────┘
└──────────┬───────────────────┘ │ │
└──────────────┬───────────────┘ │ ▼
┌──────────────────────────────┐ │ INTEGRATION ENGINE │ │ τ'(t)
= kω(t)(1 + λC(t)) │ │ τ(t) = ∫ τ'(s) ds │ │ Over window W(t) │
└──────────┬───────────────────┘ │ ┌─────────────┼─────────────┐
│ │ │ ▼ ▼ ▼ ┌─────────────┐ ┌────────┐ ┌─────────────┐ │
SUBJECTIVE │ │ SYSTEM │ │ BEHAVIORAL │ │ TIME │ │ TIME │ │
OUTPUT │ │ (Humans, │ │ (AI) │ │ (Timing │ │ Animals) │ │ │ │
Tasks) │ └─────────────┘ └──────────┘ └─────────────┘
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Figure 1: Unified Recursive Time Architecture showing the three-engine system that generates subjective and system-internal time across biological and artificial systems.

9. Evolutionary Origins and Sleep Dynamics

9.1 Why Would Evolution Produce Recursive Time?

Three selective pressures:

1. **Prediction & Survival**: Anticipate predator movements, synchronize to prey behavior, time foraging optimally, navigate seasonal cycles
2. **Metabolic Efficiency**: Minimize energy expenditure, optimize rest/activity cycles, coordinate circadian processes
3. **Social Synchronization**: Pack coordination, migration timing, mating cycles, collective vigilance

Result: Recursive time perception is an evolutionarily stable strategy (ESS) with strong selective advantages across taxa.

9.2 Sleep as Reset and Consolidation

Three functions in RTDM:

1. **Recursion Speed Reduction**: $\omega_{sleep}(t) \rightarrow \varepsilon \approx 0$ during NREM sleep
2. **Coherence Restructuring**: High-amplitude delta coherence during memory consolidation
3. **Integration Window Reset**: $\tau(t_{morning}^{+}) = \rho \cdot \tau(t_{night}^{-})$ where $0 < \rho < 0.1$

10. Limitations and Future Directions

10.1 Current Limitations

1. **Limited Direct Empirical Validation**: RTDM integrates existing literature but awaits dedicated experimental tests
2. **Parameter Estimation Challenges**: ω(t) requires combining three metrics with unknown optimal weights
3. **Model Complexity**: Seven free parameters may overfit small datasets
4. **Cross-Species Validation**: Metabolic scaling needs direct testing
5. **AI Implementation**: Optimal coherence metrics for different architectures unclear

10.2 Future Experimental Directions

Near-term (1–3 years):

- Execute five proposed experiments
- Multi-lab pre-registered replications
- Clinical translation: Temporal phenotyping in psychiatric cohorts
- Cross-species timing battery in model organisms

Medium-term (3–7 years):

- Closed-loop tACS targeting ω(t) or C(t)
- Developmental studies across lifespan
- AI engineering applications in distributed systems
- Integration with predictive processing frameworks

Long-term (7+ years):

- Bridge to physics (thermodynamic arrow, relativity)
- Societal applications (education, workplace design, space exploration)

11. Conclusion

11.1 Summary of Core Claims

We have presented the **Recursive Time Dynamics Model (RTDM)**, a unified mathematical framework for subjective and system-internal time based on three principles:

1. **Recursion speed** ($\omega(t)$) shapes the baseline rate of temporal accumulation
2. **Coherence** ($C(t)$) modulates temporal smoothness, stability, and perceived duration
3. **Periodic rhythms** ($S(t)$) organize temporal dynamics into circadian, ultradian, and computational cycles

These factors interact to produce:

$$\boxed{\tau(t) = \int_{t-W(t)}^{t} k \cdot \omega(s) \cdot (1 + \lambda C(s))\, ds}$$

a bounded, rhythm-modulated, coherence-sensitive temporal metric.

11.2 Key Innovations

1. Theoretical Unification

- First framework bridging human neuroscience, animal chronobiology, and artificial computation
- Treats time as emergent from recursion rather than external parameter

2. Quantitative Predictions

- Five falsifiable experiments with specific effect sizes
- Testable across EEG, behavior, pharmacology, tACS, and AI systems

3. Cross-Domain Applicability

- Human: ω via EEG/RT/DDM, C via PLV, S via circadian measures
- Animal: Metabolic scaling $M^{-1/4}$, entrainment coherence
- AI: Iteration rates, mutual information, clock cycles

4. Clinical Translation

- Biomarkers for depression, mania, ADHD, schizophrenia
- Treatment response prediction via parameter tracking
- Non-invasive interventions (chronotherapy, tACS, mindfulness)

11.3 Philosophical Implications

RTDM reframes time perception from a **mystery** to an **emergent computational property**:

- **Not a universal clock**: Each system generates its own time
- **Not external dimension**: Time intrinsic to recursive dynamics
- **Not phenomenological universal**: Structural homology without experiential identity
- **Not metaphysical**: Testable predictions grounded in neurobiology and engineering

11.4 Closing Statement

Time, as experienced by biological organisms and computed by artificial systems, emerges from the dynamics of recursion under constraints of coherence and rhythm. The Recursive Time Dynamics Model offers a unified, testable, mathematically rigorous framework for understanding this structure across humans, animals, and machines.

This is not a claim about universal consciousness or phenomenological identity. It is a claim about **universal mathematical structure** in systems that recursively monitor and update their internal states.

> ***Time is not a container we inhabit. Time is the sand we generate through recursion.***

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Competing Interests

The authors declare no competing financial or non-financial interests.

Data Availability

This theoretical paper does not generate new empirical data. All cited data are available in the referenced publications.

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