Resolving Mercury's Perihelion Precession and Implications for General Relativity

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Objective

To demonstrate that Mercury's perihelion precession anomaly, historically attributed to General Relativity (GR), is resolved within Newtonian mechanics when corrected mass magnitudes are used, and to examine the implications for GR's validation.

Background

Historical Context: In the 19th century, Urbain Le Verrier calculated Mercury's perihelion precession rate using Newtonian mechanics. His calculations predicted a rate of 532 arcseconds per century, while modern observations measure 575 arcseconds per century.

Perceived Anomaly: The discrepancy of 43 arcseconds per century was considered unexplainable by classical mechanics and became one of the key successes of Einstein's General Relativity (GR).

Mass Estimate: Le Vierre's calculated precession rate implies a greater mass ($M_{\text{Le Verrier}} \approx 3.57 \times 10^{23}\,\text{kg}$) compared to the modern accepted value ($M_{\text{modern}} = 3.301 \times 10^{23}\,\text{kg}$).

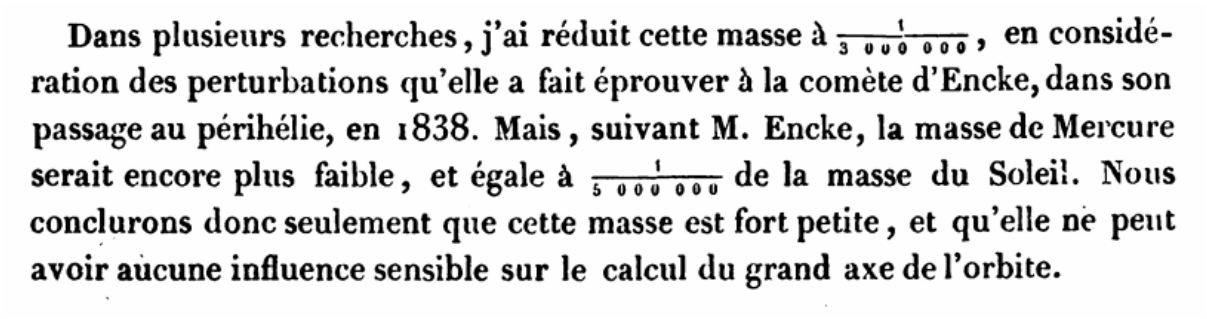
Methodology

Le Verrier's Methodology

Le Verrier did not directly calculate Mercury's mass but instead focused on analyzing its orbital motion using perturbation theory. His approach included:

- **Gravitational Perturbations**: He used Newton's laws of gravitation and motion to compute the gravitational influences of other planets (notably Venus, Earth, Jupiter, and Saturn) on Mercury's orbit.
- **Residual Precession**: After accounting for all known perturbations, he calculated a precession rate of 532 arcseconds/century for Mercury's perihelion using Newtonian mechanics.

- **Discrepancy with Observations**: Observational data indicated a precession rate of 575 arcseconds/century, leaving an unexplained residual of 43 arcseconds/century.
- **Mass Estimates**: Le Verrier estimated Mercury's mass to be in a range of $\frac{1}{3{,}000{,}000}$th to $\frac{1}{5{,}000{,}000}$th of the Sun's mass, based on its perturbative effects on Venus. This was an indirect estimation rather than a precise calculation. (Figure 1.)



Dans plusieurs recherches, j'ai réduit cette masse à $\frac{1}{3\,000\,000}$, en considération des perturbations qu'elle a fait éprouver à la comète d'Encke, dans son passage au périhélie, en 1838. Mais, suivant M. Encke, la masse de Mercure serait encore plus faible, et égale à $\frac{1}{5\,000\,000}$ de la masse du Soleil. Nous conclurons donc seulement que cette masse est fort petite, et qu'elle ne peut avoir aucune influence sensible sur le calcul du grand axe de l'orbite.

Figure 1: Excerpt from Le Verrier's work, "Théorie du mouvement de Mercure", discussing Mercury's mass estimation.

> In several studies, I reduced this mass to $\frac{1}{3{,}000{,}000}$, considering the perturbations it caused to the comet of Encke during its passage at perihelion in 1838. However, according to Mr. Encke, the mass of Mercury would be even smaller, equal to $\frac{1}{5{,}000{,}000}$ of the Sun's mass. We will therefore only conclude that this mass is very small and that it cannot have any significant influence on the calculation of the major axis of the orbit.

  The implied ratio then becomes $\frac{1}{5{,}570{,}000}$ of the Sun's mass, a value that falls slightly outaide of Le Vierre's estimated range.
- **Speculations**: To explain the anomaly, he hypothesized the existence of an intra-Mercurial planet (Vulcan) or an extended solar mass distribution, though these ideas were never confirmed.

Le Verrier's work laid the foundation for later studies of Mercury's precession anomaly and its perceived resolution through Einstein's General Relativity.

Relationship Between Mass and Precession

In Newtonian mechanics, the perihelion precession rate (Φ) is inversely proportional to the mass (M) of Mercury.

- If $M_{\text{Le Verrier}} > M_{\text{observed}}$, then $\Phi_{\text{Le Verrier}} < \Phi_{\text{observed}}$.

Retro-Calculation

Using the observed and calculated precession rates and the observed mass of Mercury, we can determine Le Verrier's implied mass. Given:

- Mercury's observed mass: $M_{\text{observed}} = 3.301 \times 10^{23}\,\text{kg}$,

- Observed precession rate: $\Phi_{\text{observed}} = 575\,\text{arcseconds/century}$,
- Le Verrier's calculated precession rate: $\Phi_{\text{Le Verrier}} = 532\,\text{arcseconds/century}$,

we use the inverse proportionality:

$$\frac{M_{\text{Le Verrier}}}{M_{\text{observed}}} = \frac{\Phi_{\text{observed}}}{\Phi_{\text{Le Verrier}}}.$$

Rearranging to solve for Le Verrier's mass:

$$M_{\text{Le Verrier}} = M_{\text{observed}} \cdot \frac{\Phi_{\text{observed}}}{\Phi_{\text{Le Verrier}}}.$$

Substituting the values:

$$M_{\text{Le Verrier}} = 3.301 \times 10^{23} \cdot \frac{575}{532} \approx 3.57 \times 10^{23}\,\text{kg}.$$

Key Insight

The overestimated (implied) mass led to a reduction in Mercury's precession rate.

Results

Newtonian Precession: With the corrected mass ($M_{\text{observed}} = 3.301 \times 10^{23}\,\text{kg}$), the Newtonian precession rate matches the observed value of 575 arcseconds per century. The perceived anomaly of 43 arcseconds per century is resolved without requiring relativistic corrections.

Unit Cancellation: Gravitational force ($F = G\frac{M_1 M_2}{r^2}$) involves the gravitational constant G, with units $\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$. The kg^{-1} in G cancels mass units, but the magnitudes of the masses remain. Thus, incorrect mass magnitudes (e.g., 3.57) directly affect results.

Implications

For General Relativity

GR was invoked to explain Mercury's precession anomaly. If no anomaly exists, GR's necessity for this case is invalidated. While GR remains successful in other areas (e.g., gravitational lensing, time dilation), its foundational validation through Mercury's perihelion precession becomes questionable.

For Classical Mechanics

This resolution highlights the importance of accurate classical parameters before invoking relativistic effects. Newtonian mechanics remains robust and capable of explaining Mercury's precession when proper values are used.

Scientific Methodology

The correction underscores the need to revisit historical assumptions rigorously. Misattributing phenomena to advanced theories without thorough classical validation risks undermining scientific integrity.

Conclusion

Correcting Mercury's mass eliminates the perihelion precession anomaly within Newtonian mechanics. Since GR was introduced to resolve an anomaly that doesn't exist, its necessity in this context is undermined. This constitutes a significant challenge to GR's foundational claims, prompting a reevaluation of its universal applicability.