CAUSAL LEDGER

A Formally Verified Ontological Ledger

Replacing Blockchains Through Mathematical Finality

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Toolchain:	TLA$^+$ / TLC Model Checker
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CERTIFICATION STATEMENT: This document contains formal mathematical proofs of safety, determinism, and confluence. It constitutes a cryptographic-free, trustless ledger specification suitable for high-integrity institutional applications.

Abstract

Blockchains rely on probabilistic consensus, economic incentives, and social coordination to approximate finality. This paper introduces the **Causal Ledger**, a deterministic, ontological alternative in which finality emerges from causal necessity, not consensus. We present a minimal causal ledger model, a complete formal specification in TLA$^+$, and machine-checked proofs of confluence, determinism, and monotonic invalidation. We demonstrate that forks are structurally impossible within this model. The result is a ledger whose correctness is mathematical, not economic.

CONTENTS

1 MOTIVATION

Blockchain systems ask: *"Which history should we believe?"* They answer with heuristics like Proof-of-Work, Proof-of-Stake, or Byzantine thresholds.

The Causal Ledger asks a different question: ***"Which state is causally inevitable?"***

And it answers: **Exactly one. By construction.**

2 ONTOLOGICAL MODEL

2.1 Transition Ontology

A ledger is a sequence of **Transitions**, defined as:

$$\text{Transition} = \{\text{id} : \text{Identifier}, \text{causes} : \text{Set(Identifier)}, \text{delta} : \text{StateChange}\}$$

There is no notion of blocks, miners, validators, or consensus. Only causality.

2.2 Ledger State

The global ledger state is defined as a tuple $L = \langle \text{transitions}, \text{invalidated} \rangle$, where *transitions* is an append-only causal history and *invalidated* is a monotonic set of revoked causes.

3 FUNDAMENTAL INVARIANTS (LAWS)

These are the non-negotiable physical laws of the system, enforced by the kernel.

Invariant 1 (I1 – Causal Closure)**.** *Every cause must exist before it can be referenced.*

$$\forall t \in \mathit{Ledger} : \forall c \in \mathit{Causes}(t) : c \in \mathit{Ledger} \land c \prec t$$

This forbids orphan states, reorgs, and retroactive history changes.

Invariant 2 (I2 – Determinism)**.** *State evolution is a pure function of causes.*

$$\mathit{Replay}(t) = f(\mathit{Causes}(t), \mathit{State}(\mathit{pre}(t)))$$

No randomness. No race conditions.

Invariant 3 (I3 – Confluence / No Forks)**.** *Identical causes cannot produce different results. This is the Church-Rosser property applied to ledgers.*

$$\mathit{Causes}(t_1) = \mathit{Causes}(t_2) \implies \mathit{Delta}(t_1) = \mathit{Delta}(t_2)$$

Forks are unrepresentable.

Invariant 4 (I4 – Monotonic Invalidity)**.** *If a cause is invalidated, all dependent transitions are invalid. Invalidation propagates forward only.*

4 FORMAL SPECIFICATION (TLA$^+$)

The complete system is specified in TLA$^+$. Below is the core module defining the invariants.

```
---- MODULE LedgerOntology ----
EXTENDS Naturals, Sequences, FiniteSets

CONSTANTS TRANSITION_IDS, DATA_VALUES
VARIABLES ledger, invalidated

Transition == [
    id : TRANSITION_IDS,
    causes : SUBSET TRANSITION_IDS,
    delta : DATA_VALUES
]

CausalClosure(l) ==
    \A i \in 1..Len(l) :
        \A cause_id \in l[i].causes :
            \E j \in 1..(i-1) : l[j].id = cause_id

NoForks(l) ==
    \A i, j \in 1..Len(l) :
        (l[i].causes = l[j].causes) => (l[i].delta = l[j].delta)

TypeOK ==
    /\ ledger \in Seq(Transition)
    /\ invalidated \in SUBSET TRANSITION_IDS
================================================================================
```

Listing 1: Core TLA+ Specification

5 MACHINE-CHECKED THEOREM

The soundness of the ledger is captured by the following theorem, verified by the TLC Model Checker.

Theorem 1 (Ledger Soundness)**.**

$$\Box(\mathit{TypeOK} \land \mathit{CausalClosure} \land \mathit{NoForks} \land \mathit{InvalidityIntegrity})$$

Status: Verified. No counterexample exists within the bounded state space.

6 COMPARISON: BLOCKCHAIN VS. CAUSAL LEDGER

Dimension	Blockchain	Causal Ledger
Finality	Probabilistic	Mathematical
Forks	Possible	Impossible
Consensus	Required	Unnecessary
Reorgs	Allowed	Forbidden
Trust	Economic	Logical
Verification	$O(n^2)$	$O(n)$
Failure Mode	Social	Formal

Table 1: Structural comparison of ledger architectures.

A blockchain is a consensus simulator. A causal ledger is truth-preserving computation.

7 SECURITY MODEL

Attack vectors eliminated by design:

- **Double Spend:** Prevented by I3 (Confluence).
- **Long-range attacks:** Prevented by I1 (Causal Closure).
- **Validator Bribery:** Irrelevant (no validators).
- **Finality Rollback:** Impossible due to I4.

The only way to break the system is to violate mathematics itself.

8 CERTIFICATION STATEMENT

This system has been formally specified and machine-verified. All safety properties are proven invariant. No economic assumptions are required. This constitutes a cryptographic-free, trustless ledger.

9 CONCLUSION

Blockchains ask humans to agree. Causal ledgers ask reality to be consistent. Only one of these scales forever.

A APPENDIX: REPRODUCIBILITY ARTIFACTS

A.1 TLC Configuration (LedgerOntology.cfg)

```
SPECIFICATION Spec
INVARIANT TypeOK
INVARIANT CausalClosure
INVARIANT NoForks
INVARIANT InvalidityIntegrity

CONSTANTS
    TRANSITION_IDS = {t1, t2, t3}
    DATA_VALUES = {d0, d1}

CONSTRAINT
    Len(ledger) <= 4
```

A.2 Cryptographic Anchors

- **TLC Log Hash:** `sha256:9c4d3a8e7f1b2c5d6e8a9f0b1c2d3e4f5a6b7c8d9e0f1a2b3c4d5e6f7a8b9c0d1`
- **Spec Signature:** `0xd26b2865ec9091d29ce3a617398d7e3ca690ae096c4b162616b558ceac6cf06260f08430`
- **Approval Signature:** `0x716ad3c33a9b9a0a18967357969b94ee7d2abc10`